PHYS 6610: Graduate Nuclear and Particle Physics I

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II. Phenomena

1. Shapes and Masses of Nuclei

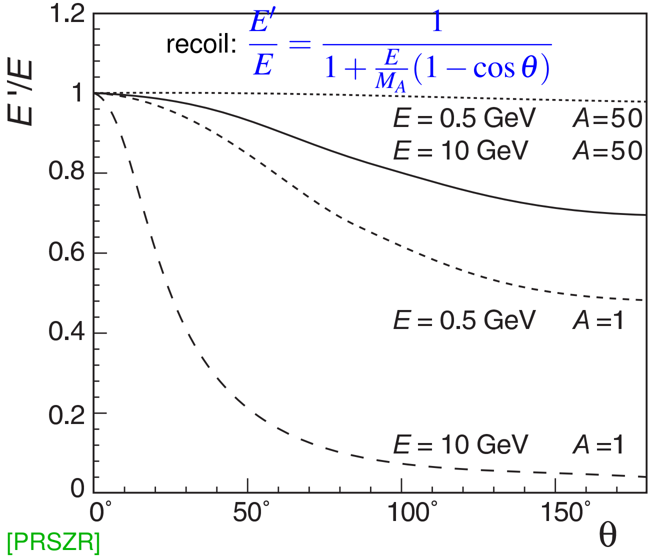
Or: Nuclear Phenomeology

References: [PRSZR 5.4, 2.3, 3.1/3; HG 6.3/4, (14.5), 16.1; cursorily PRSZR 18, 19]

(a) Getting Experimental Information

heavy, spinless, composite target $\Longrightarrow M \gg E' \approx E \gg m_e \to 0$

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{lab}} = \underbrace{\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}}_{\text{Mott: spin-}\frac{1}{2}\text{ on spin-}0,\ m_e=0,\ M_A\neq 0} \frac{E'}{E} \left|F(\vec{q}^{\,2})\right|^2 \qquad \text{(I.7.3)}$$



[PRSZR]

when nuclear recoil negligible: $M_A \gg E \approx E'$

$q^2 = (k-k')^2 \to -\vec{q}^{\,2} = -2E^2(1-\cos\theta)$

translate $q^2 \Longleftrightarrow \theta \Longleftrightarrow E$

max. mom. transfer: $\theta \to 180°$: $q^2 \to -(2E)^2$

min. mom. transfer: $\theta \to 0°$: $q^2 \to 0$

For $E = 800$ MeV (MAMI-B), ^{12}C ($A = 12$):

θ	$\sqrt{-q^2} = \lvert\vec{q}\rvert$	$\Delta x = \frac{1}{\lvert\vec{q}\rvert}$
180°	1500 MeV	0.15 fm
90°	1000 MeV	0.2 fm
30°	400 MeV	0.5 fm
10°	130 MeV	1.5 fm

includes E/M (recoil) effects

"Typical" Example: ^{40,48}Ca measured over 12 orders

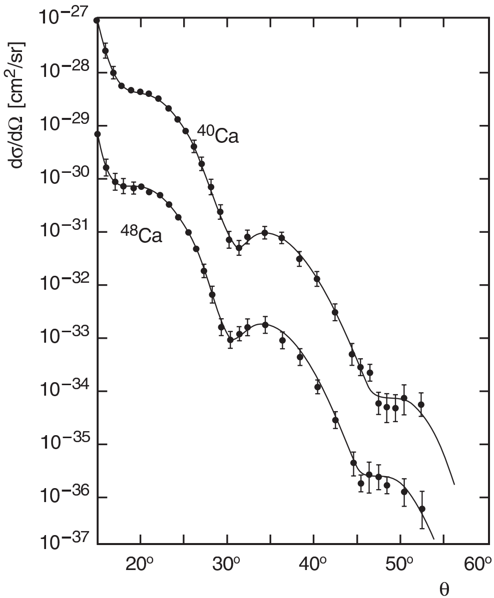
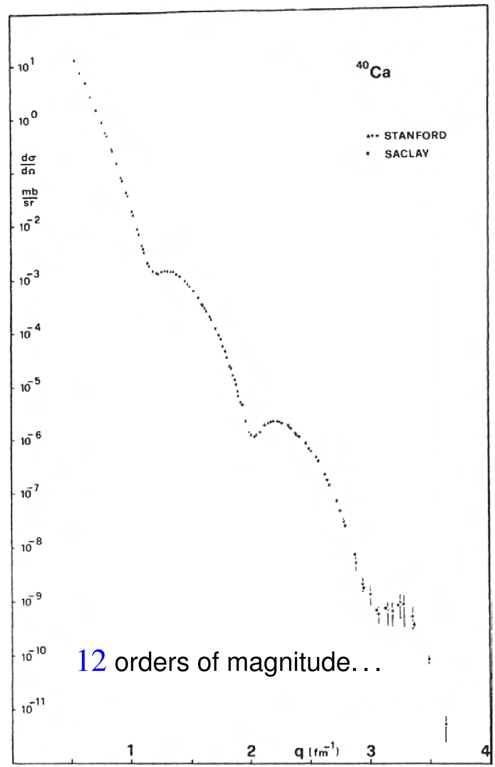


Fig. 5.7. Differential cross-sections for electron scattering off the calcium isotopes ^{40}Ca and ^{48}Ca [Be67]. For clarity, the cross-sections of ^{40}Ca and ^{48}Ca have been multiplied by factors of 10 and 10^{-1}, respectively. The solid lines are the charge distributions obtained from a fit to the data. The location of the minima shows that the radius of ^{48}Ca is larger than that of ^{40}Ca.

θ-dependence, multiplied by $10, 0.1$!! [PRSZR]

^{40}Ca has less slope $\Longrightarrow$ smaller size



$|\vec{q}|$-dependence fm^{-1} [HG 6.3]

Characterising (Spherically Symmetric) Charge Densities

$$F(\vec{q}^2) := \frac{4\pi}{Ze} \int\limits_0^\infty \mathrm{d}r \, \frac{r}{q} \, \sin(qr) \, \rho(r), \quad \text{normalisation } F(\vec{q}^2 = 0) = 1$$

In principle: Fourier transformation $\Longrightarrow$ need $F(\vec{q}^2 \to \infty)$: impossible

$\Longrightarrow$ **Ways out:**

(i) Characterise just object size:

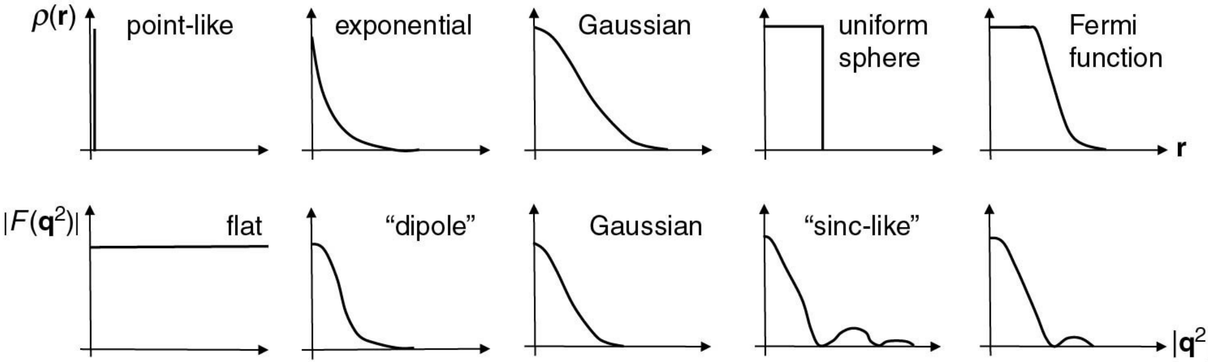
$$F(\vec{q}^2 \to 0) = \frac{4\pi}{Ze} \int\limits_0^\infty \mathrm{d}r \, \frac{r}{q} \overbrace{\left[qr - \frac{(qr)^3}{3!} + \mathcal{O}((qr)^5) \right]}^{\sin qr} \rho(r)$$

$$= \underbrace{\frac{4\pi}{Ze} \int\limits_0^\infty \mathrm{d}r \, r^2 \, \rho(r)}_{=1} - \frac{q^2}{3!} \underbrace{\frac{4\pi}{Ze} \int\limits_0^\infty \mathrm{d}r \, r^2 \times r^2 \, \rho(r)}_{\text{mean of } r^2 \text{ operator}} + \mathcal{O}((qr)^5)$$

$$\Longrightarrow \langle r^2 \rangle = -3! \, \left. \frac{\mathrm{d}F(\vec{q}^2)}{\mathrm{d}\vec{q}^2} \right|_{\vec{q}^2=0}$$

(square of) **root-mean-square (rms) radius**

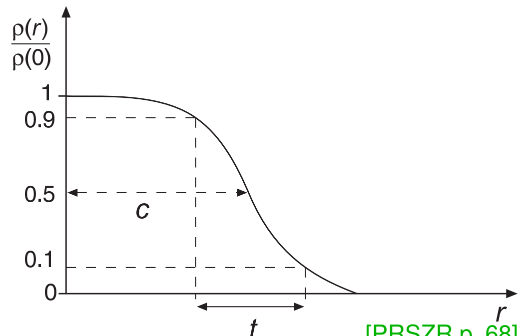
Ways Out: (ii) Assume Charge Density, Calculate its FF, Fit



[Tho 7.5] Dirac fermion | proton | ^{6}Li nucleus | 1st min at $qR \approx 4.5$ | ^{40}Ca nucleus

Very simple form for heavy nuclei: **Fermi Distribution**

matter density $\rho_N(r) = \dfrac{\rho_0}{1+\exp\frac{r-c}{a}} = \dfrac{A}{Z}\,\rho_{\text{charge}}$



[PRSZR p. 68]

experiments: radius at half-density $c \approx A^{1/3} \times 1.07\ \text{fm}$:

translate hard-sphere: $R = \sqrt{\frac{5}{3}\langle r^2\rangle} \approx A^{1/3} \times 1.21\ \text{fm}$

experiments: $a \approx 0.5\ \text{fm}$: largely independent of A!;

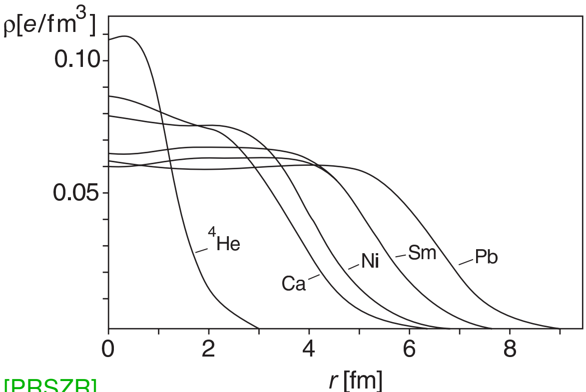
related to surface/skin thickness $t = a \times 4\ln 3 \approx 2.40\ \text{fm}$ (drop from 90% to 10%)

Charge and Matter Densities in Nuclei

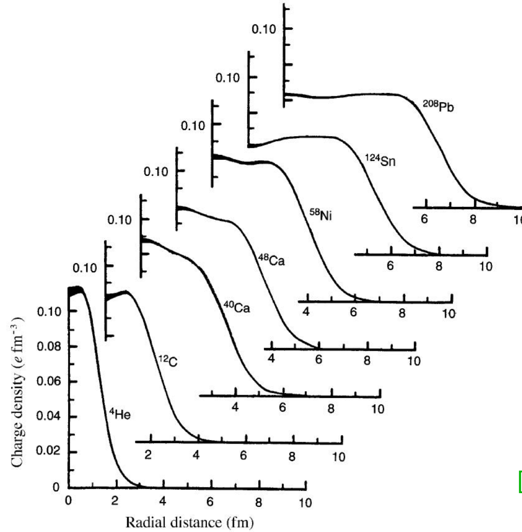
Better parameterise as sum of a few Gauß'ians:

$$\rho_{\text{charge}}(r) = \sum_i b_i \exp - \frac{(r - R_i)^2}{\delta^2}$$

$\Longrightarrow$ "line thickness" = parametrisation uncertainty



[PRSZR]



[Mar]

Figure 2.4 Radial charge distributions ρ_{ch} of various nuclei, in units of e fm^{-3}; the thickness of the curves near $r = 0$ is a measure of the uncertaintity in ρ_{ch} (adapted from Fr83)

$\rho_{\text{charge}}(r=0)$ decreases with A, but matter density $\rho_N = \frac{A}{Z}\,\rho_{\text{charge}}(r=0)$ constant for large A:

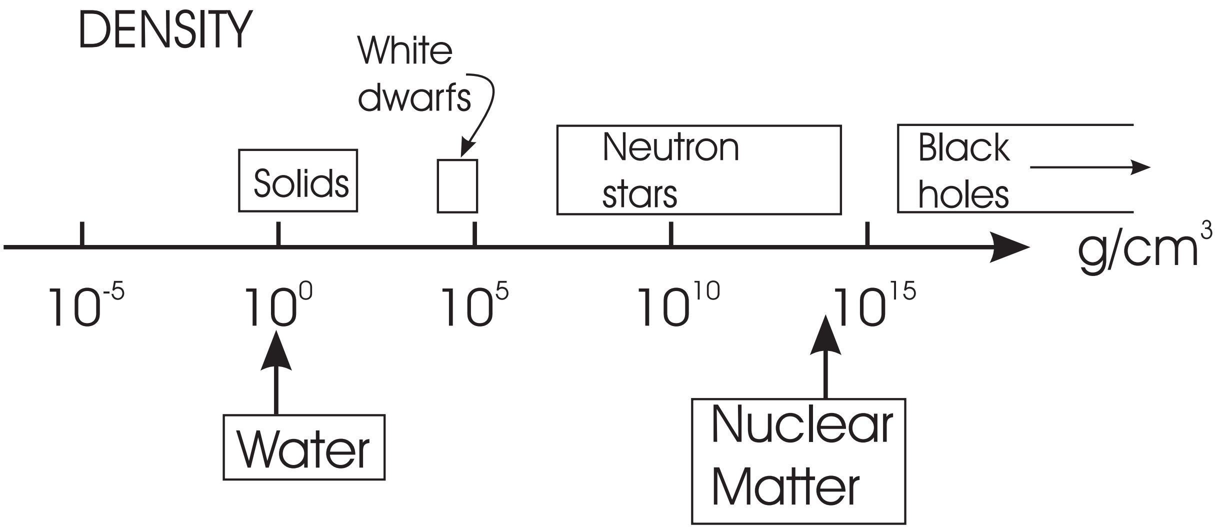
ρ_N **plateaus in heavy nuclei** $\Longrightarrow$ **Saturation of Interaction**: attraction long-range, repulsion up-close!?!

Nucleons "separate but close": Distance between nucleons in nucleus $\approx 1.4\times$ nucleon rms diameter.

$$\rho_N \approx \frac{0.17 \text{ nucleons}}{\text{fm}^3} \approx \frac{1}{(1.4 \times (2 \times 0.7\text{fm}))^3} \,\widehat{=}\, 3 \times 10^{11}\,\frac{\text{kg}}{\text{litre}} = 300\,\frac{\text{tonnes}}{\text{mm}^3} = \frac{3 \text{ space shuttles}}{\text{mm}^3}$$

Putting ρ_{N} In Perspective

$$\rho_N \approx \frac{0.17\ \text{nucleons}}{\text{fm}^3} \approx \frac{1}{(1.4\times(2\times 0.7\text{fm}))^3} \mathrel{\widehat{=}} 3\times 10^{11}\ \frac{\text{kg}}{\text{litre}} = 300\ \frac{\text{tonnes}}{\text{mm}^3} = \frac{3\ \text{space shuttles}}{\text{mm}^3}$$



[HG fig 1.3]

(b) Spin & Deformation Example: Deuteron [HG 14.5; [nucl-th/0608036]; [nucl-th/0102049]]

So far: no spin, no deformation, **but** spherically-symmetric $F(\vec{q}^{\,2})$ is the exception, not the rule.

Deformation Example Deuteron $d(np)$ with $J^{PC} = 1^{+-}$: $\vec{L} = \vec{J} - \vec{S}$, $S = 1$

$$L = 1_J \otimes 1_S = 0 \oplus 1 \oplus 2 \implies L = 0 \text{ (s-wave) or } L = 2 \text{ d-wave – Parity forbids } L = 1.$$

angular momentum $\implies$ mag. moment $\implies$ spin-spin interaction/spin transfer ($\theta \to 180°$, helicity)

Charge FF: $G_C(q^2) = \dfrac{e}{3} \sum\limits_{m_J=-1}^{1} \langle m_J | J^0 | m_J \rangle$ avg. of hadron density, $G_C(0) = 1$

Magnetic FF: $G_M(q^2)$ $G_M(0) =: \dfrac{M_d}{M_N}\, \kappa_d$, mag. moment $\kappa_d = 0.857$

Quadrupole FF: $G_Q(q^2)$

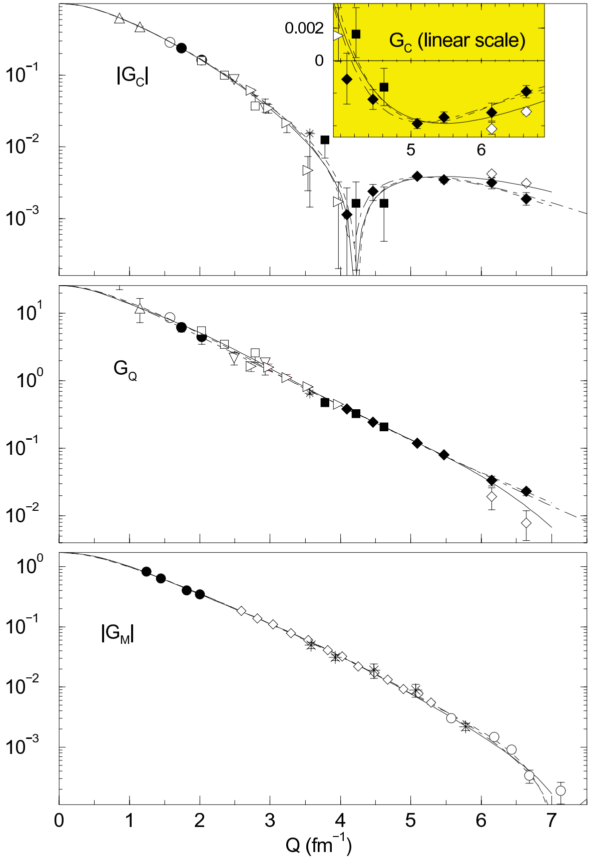
quadrupole moment $Q_d := G_Q(0) = Ze \int \mathrm{d}^3 r \, \overbrace{(3z^2 - r^2)}^{\text{quadrupole op.}} \, \rho_{\text{charge}}(\vec{r}) = 0.286 \text{ fm}^2$

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{lab}} = \underbrace{\left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}}_{\text{Mott}} \times \left[\left(G_C^2 + \frac{2\tau}{3} G_M^2 + \frac{8\tau^2 M_d^4}{9} G_Q^2\right) + \underbrace{\frac{4\tau(1+\tau)}{3} G_M^2 \tan^2\frac{\theta}{2}}_{\text{helicity} \implies \text{spin transfer}}\right]$$

with $\tau = -\dfrac{q^2}{4M_d^2}$ as before

$\implies$ Dis-entangle by θ & τ dependence.

Deuteron Form Factors



[Garçon/van Orden: [nucl-th/0102049]]

does not include most recent data (JLab),

but pedagogical plot

high-accuracy data up to $Q^2 \gtrsim (1.4\ \mathrm{GeV})^2$

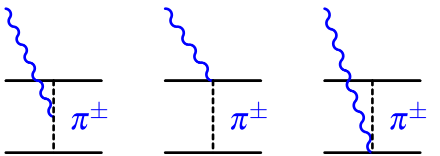
Theory quite well understood:

embed nucleon FFs into deuteron, add:

deuteron bound by short-distance

$+$ tensor force: one-pion exchange $N^\dagger \vec{\sigma} \cdot \vec{q}\,\pi(q) N$

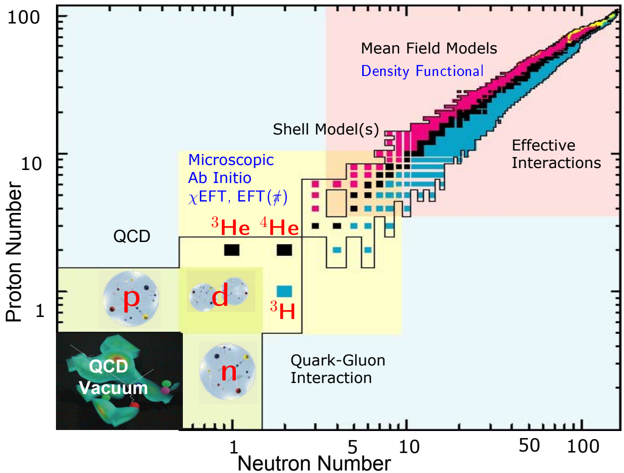
$\Longrightarrow$ photon couples to charged meson-exchanges

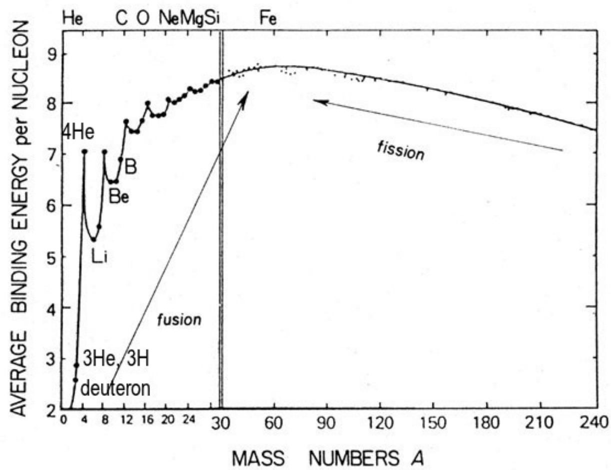


and many more!!

open issues: zero of G_Q, form for $Q \gtrsim 3\ \mathrm{fm}^{-1}$,...

(c) Nuclear Binding Energies (per Nucleon)





B/A	rapidly increases from deuteron ($A = 2$):	$1.1\ \text{MeV}/A$
	to about ^{12}C:	$7.5\ \text{MeV}/A$
	for $A \gtrsim 16$ (oxygen) remains around	$7.5\ldots8.5\ \text{MeV}/A$
	maximal for ^{56}Fe–^{60}Co–^{62}Ni:	$8.5\ \text{MeV}/A$
	small decrease to $A \approx 250$ (U):	$7.5\ \text{MeV}/A$

$\Longrightarrow$ "typically", fusion gains (much) energy up to Fe; fission gains (some) energy after Fe.

$\Longrightarrow$ Fe has relatively large abundance: product of both exothermal fusion and fission.

Bethe-Weizsäcker Mass Formula & Interpretation: Liquid Drop Model

Know < 3000 nuclei $\Longrightarrow$ *roughly* parametrise *ground-state* binding energies, not only for stable nuclei

Total binding energy: SEMF
Semi-Empirical Mass Formula

$$B = a_V\,A - a_s\,A^{2/3} - a_C\,\frac{Z^2}{A^{1/3}} - a_a\,\frac{(N-Z)^2}{4A} - \frac{\delta}{A^{1/2}}$$ (1935/36)

$a_V = 15.67$ MeV — **volume term** — cf. $\rho_N \approx 0.17\ \text{fm}^{-3} \Longrightarrow$ **saturation**

$\Longrightarrow$ ***Well-separated, quasi-free nucleons, next-neighbour interactions like in liquid.***

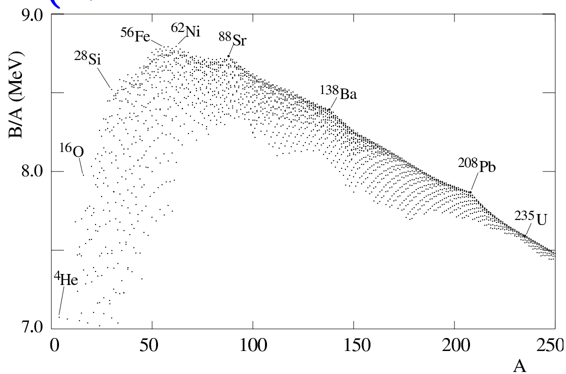
$a_s = 17.23$ MeV — **surface tension** — fewer neighbours on surface $\Longrightarrow$ less B

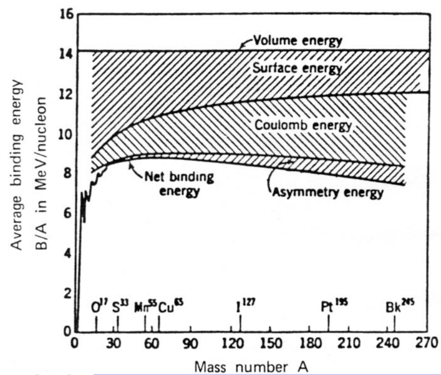
$a_C = 0.714$ MeV — **Coulomb repulsion** — of protons $\Longrightarrow$ tilt to $N > Z$

$a_a = 93.15$ MeV — **(a)symmetry/Pauli term** — Pauli principle $\Longrightarrow$ tilt to $N \sim Z$

$$\delta = \begin{cases} -11.2\ \text{MeV} & Z\ \&\ N \text{ even} \\ 0 & Z \text{ or } N \text{ odd} \\ +11.2\ \text{MeV} & Z\ \&\ N \text{ odd} \end{cases}$$

pairing term — opposite spins have net attraction; wf overlap decreases with A $\Longrightarrow$ A-dependence

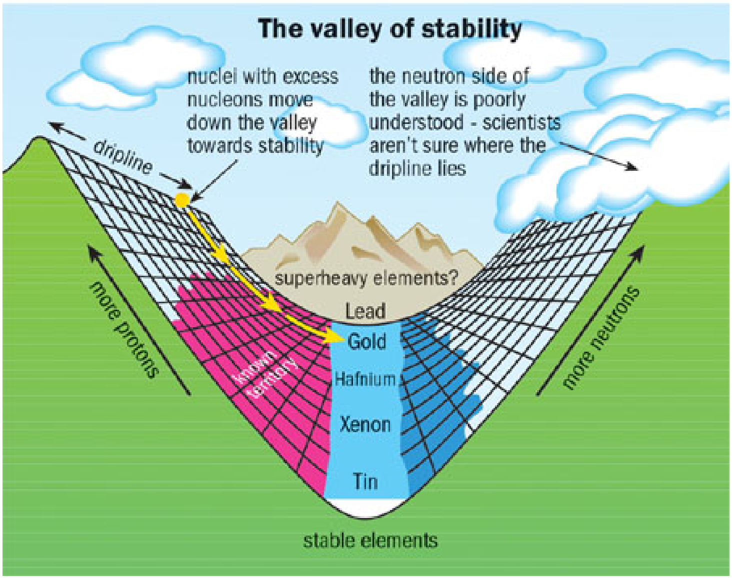


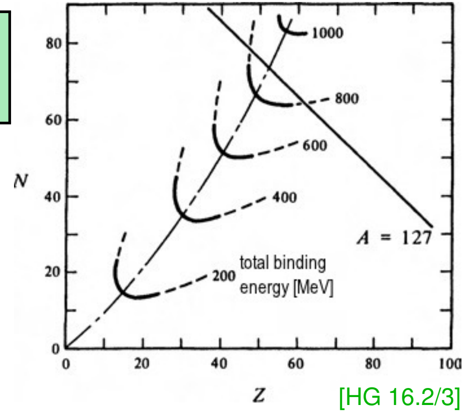


Valley of Stability around $N \gtrsim Z$

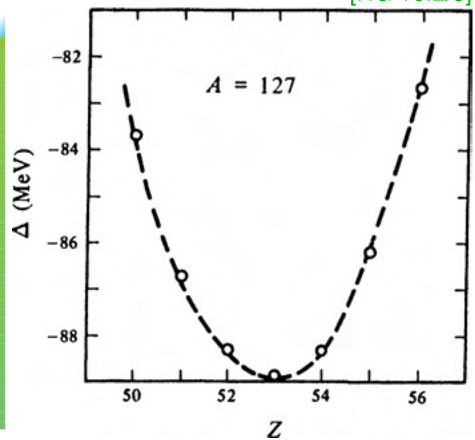
$$B = a_V\,A - a_s\,A^{2/3} - a_C\,\frac{Z^2}{A^{1/3}} - a_a\,\frac{(A-2Z)^2}{4A} - \frac{\delta}{A^{1/2}}$$

$\Longrightarrow$ Parabola in Z at fixed A with $Z_{\min} = \dfrac{a_a A}{2(a_a + a_C A^{2/3})} \lesssim \dfrac{A}{2}$





[HG 16.2/3]

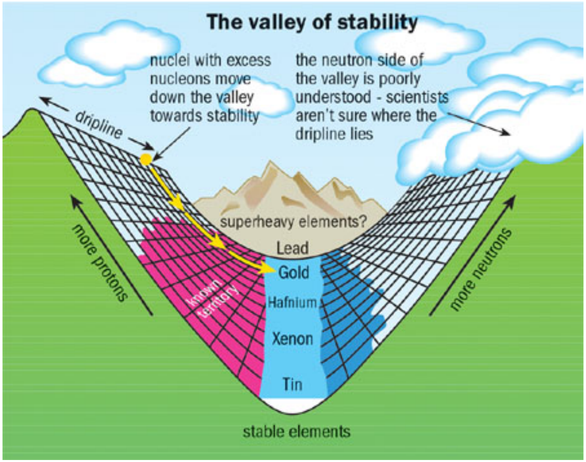


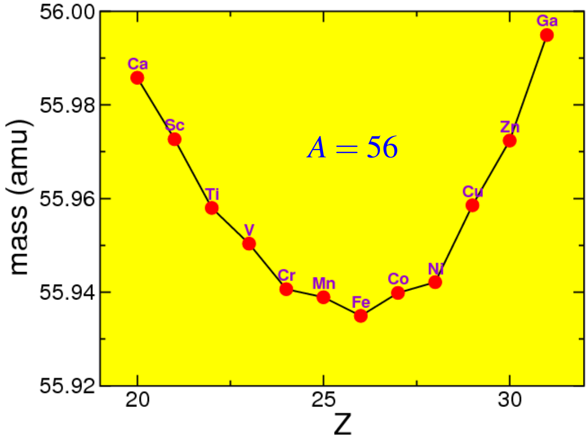
Valley of Stability

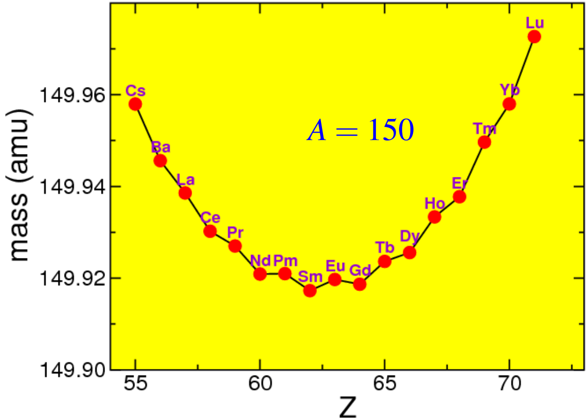
Probe nuclear interactions

by pushing to **"drip-lines"**:

Facility for Rare Isotope Beams FRIB at MSU

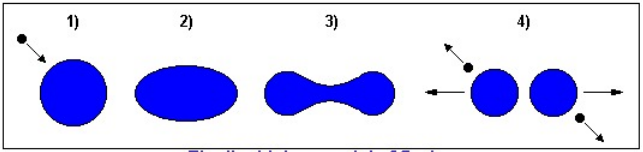






(d) Application: Nuclear Fission

[PRSZR 3.3]



The liquid drop model of fission.

For $A > 60$, fission can release energy,

but must overcome **fission barrier**.

Let's assume fission into 2 equal fragments.

Estimate: infinitesimal deformation into ellipsoid (egg) with excentricity ϵ at constant volume

$\Longrightarrow$ surface tension $\nearrow$, Coulomb $\searrow$:

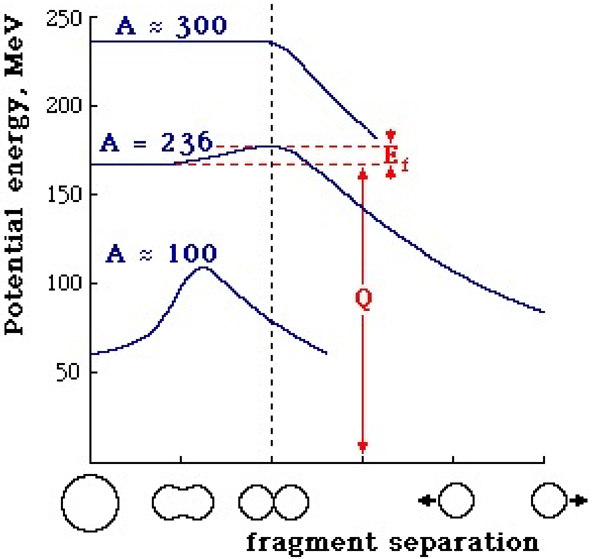
$$E(\text{sphere}) - E(\text{ellipsoid}) \sim \frac{\epsilon^2}{5}\left(2a_s A^{2/3} - a_C Z^2 A^{-1/3}\right)$$

Fission barrier classically overcome when ≤ 0:

$$\frac{Z^2}{A} \sim \frac{2a_s}{a_C} \approx 48 \qquad \text{e.g. } Z > 114,\ A > 270$$

below: QM tunnel prob. $\propto \exp -2\int\sqrt{2M(E-V)}$

between points with $r(E=V)$



Induced Fission: importance of pairing energy

$n + {}^{238}_{92}U \rightarrow {}^{239}_{92}U$: even-even $\rightarrow$ even-odd

$\Longrightarrow$ **invest** pairing $E_\delta = \frac{\delta = 11.2\text{MeV}}{\sqrt{239}} = 0.7\text{MeV}$

$n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U$: even-odd $\rightarrow$ even-even

$\Longrightarrow$ **gain** pairing energy $\frac{\delta}{\sqrt{236}} = -0.7\text{MeV}$ $\Longrightarrow$ can use thermal neutrons (higher σ!)

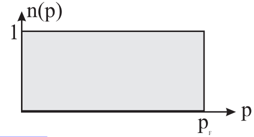
(e) First Dash into Nuclear Matter

Nuclear Interactions Saturate: $\rho_N \approx \dfrac{0.17\text{ nucleons}}{\text{fm}^3} \to$ const. in heavy nuclei

Nucleons "separate but close": Distance between nucleons in nucleus $\approx 1.4\times$ nucleon rms diameter.

Fermi distribution at temperature $T = 0$ for $N = Z$:

occupy all levels, 2 spins, proton & neutron, $N = Z$



$$\rho_N = \rho_p + \rho_n = 2 \int\limits_{|\vec{k}|\leq k_F} \frac{\mathrm{d}^3k}{(2\pi)^3}\,[n_p(\vec{k}) + n_n(\vec{k})] = \frac{2}{3\pi^2}\,k_F^3$$

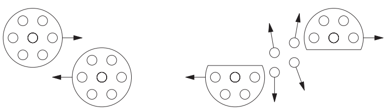
$\Longrightarrow$ **Fermi momentum** (max. nucleon momentum) $k_F = \sqrt[3]{\dfrac{3\pi^2\rho_N}{2}} \approx 1.3\text{fm}^{-1} \approx 260\text{MeV} \approx 2m_\pi$.

Liquid-gas transition for temperature $T \nearrow$

Liquid-Drop Model: heat $\Longrightarrow$ evaporation

T via E-distrib. of collision fragments:

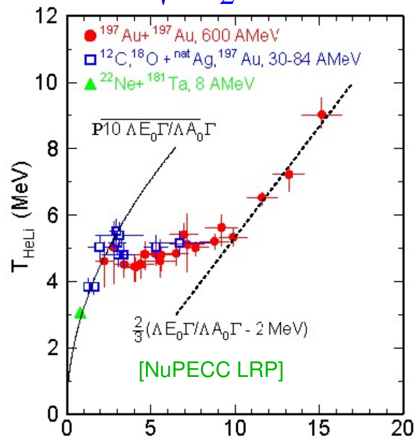
$$N(E) \propto \sqrt{E}\,\exp[-E/T] \text{ (Maxwell)}$$



but need many fragments, angle-indep.

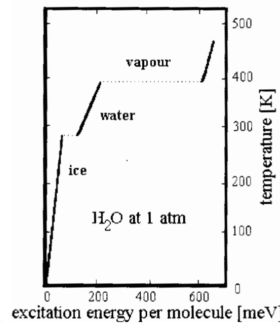
exp: $T \approx 5\text{MeV}$ in *finite* symmetric nuclei.

Neutron-E-distrib. in ^{235}U fission: $T = 1.29\text{MeV}$



excitation energy E/A [MeV] per nucleon

compare to water



Nuclei Are Not "Nuclear Matter"

SEMF of finite nuclei: $\frac{B}{A} = a_V - \frac{a_s}{A^{1/3}} - a_C \frac{Z^2}{A^{4/3}} - a_a \frac{(N-Z)^2}{4A^2} - \frac{\delta}{A^{3/2}} \approx 8.5\text{MeV}$

Infinite nuclear matter: no surface, Coulomb negligible, no pairing $\Longrightarrow \frac{B}{A} \approx a_V = 15.6\text{MeV}$.

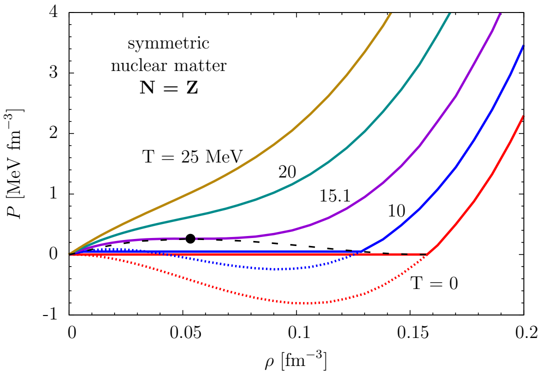
grand canonical ensemble: $\mathcal{Z} = \text{tr} \exp -\frac{1}{T}[H - \mu_p N_p - \mu_n N_n]$ with μ_N: chem. potentials

$$-T \ln \mathcal{Z} = -PV = E - TS - \mu_p N_p - \mu_n N_n$$

$\Longrightarrow$ pressure $-P = \mathcal{E} - Ts - \mu_p \rho_p - \mu_n \rho_n$ with $\mathcal{E}$: energy density, s: entropy density

Need to extrapolate or solve nuclear many-body problem: specify interactions!

Descriptions agree at $\rho_0 \approx 0.16\text{fm}^{-3}$ — here χEFT (π, N, $\Delta(1232)$) [Fiorilla/... [arXiv:1111.3688 [nucl-th]]]

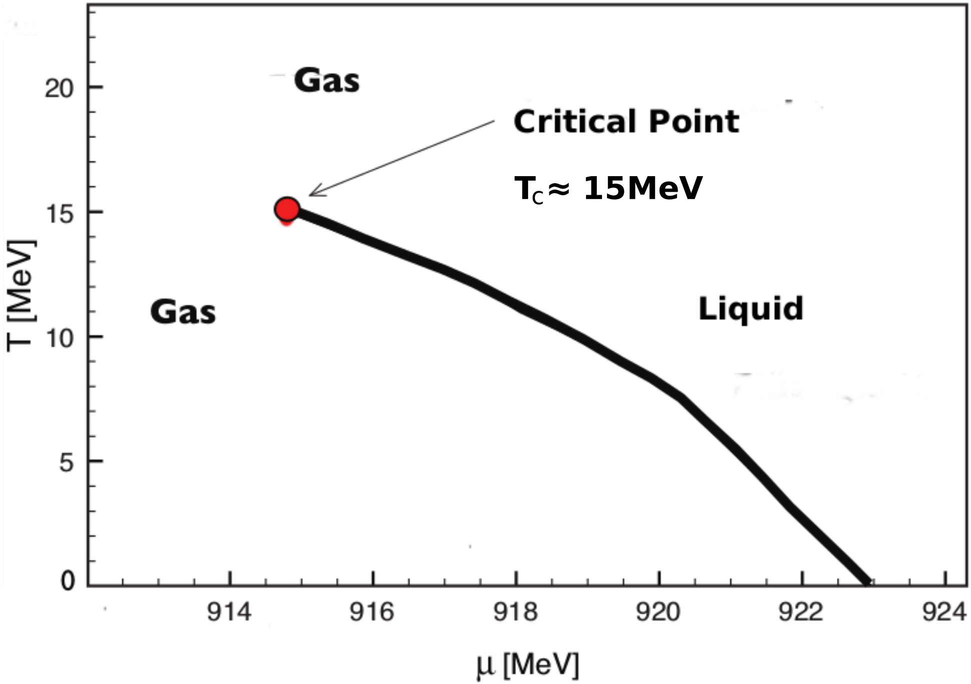


Empirical **first-order liquid-gas phase transition** for infinite, symmetric ($N = Z$) nuclear matter at critical temperature $T_c = [16 \dots 18]\text{MeV}$.

Chemical potential at temperature $T = 0$:

$$\mu_N(T=0) = M_N - \frac{B}{A} = [939 - 16]\text{MeV} \approx 923\text{MeV}$$

A First Phase Diagram of Nuclear Matter: $N = Z$

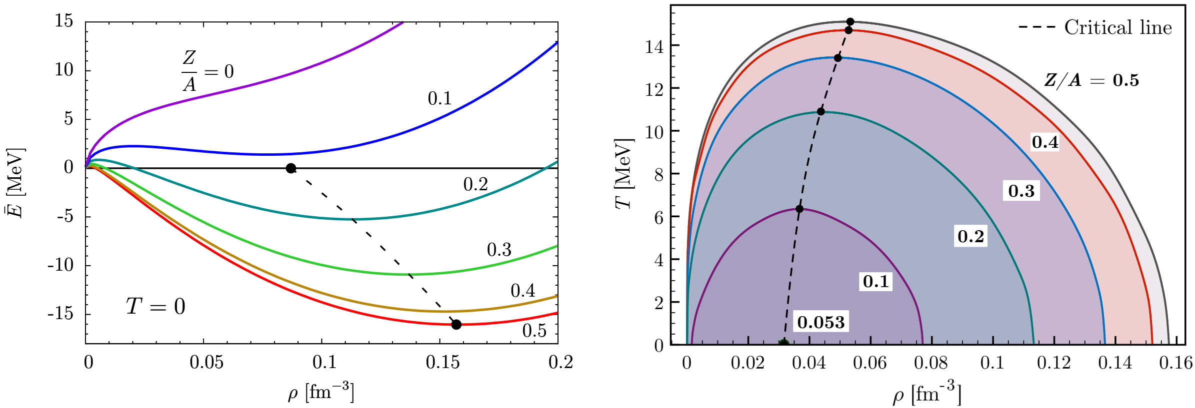


$T_c \ll m_\pi, M_N \implies$ symmetric nuclear matter close to liquid-gas transition: just inside liquid phase.

Density $\rho(T, \mu_N)$ of stable nuclear matter depends on T, μ.

Nature is Not Symmetric: Dependence on Proton-Neutron Mix

Again relatively good agreement between descriptions – here χEFT [Fiorilla/... [arXiv:1111.3688 [nucl-th]]].



$\Longrightarrow$ Nuclear-matter density $\rho(N-Z)$ decreases as Z/A decreases.

– Nuclear Matter becomes unbound for $Z/A \lesssim 0.1$.

– Why is pure-neutron matter unbound (a gas)?? (Pauli-principle??)

– In early 1900's, neutron "invented" to mitigate Coulomb repulsion between protons.

So why no binding when I take all protons away?

– Lattice QCD: Neutron matter might actually be bound for larger $m_\pi > 600$ MeV (controversial).

So Why Are There Neutron Stars??

Gravity compacts interior $\Longrightarrow$ saturation point shifts: $\rho_0 \approx 0.16\text{fm}^{-3} \to 3\rho_0$, holds neutrons together.

How to extrapolate to there – and how to extrapolate from $Z \approx 0.4A$ to $Z \lesssim 0.1A$ ("neutron" star!)?

Taylor in $\frac{N-Z}{A}$: $\mathcal{E}(\rho, \frac{N-Z}{A}) = \mathcal{E}_0(\rho_0, \frac{N-Z}{A} = 0) + \left[\frac{a_a(\rho_0)}{4} + \frac{\mathrm{d}(a_a/4)}{\mathrm{d}\rho}\Big|_{\rho_0}\right] \left(\frac{N-Z}{A}\right)^2 + \dots$

Nuclei (SEMF): "(a)symmetry energy" $a_a(\rho_0)/4 \approx 22\text{ MeV}$; nucl. matter: $[29\dots33]\text{ MeV}$

slope $L = 3\frac{\mathrm{d}(a_a/4)}{\mathrm{d}\ln\rho}\Big|_{\rho_0} = [40\dots62]\text{ MeV}$. Method: compare different Z/A nuclei & extrapolate.

Taylor in $(\rho - \rho_0)$: $\mathcal{E}(\rho, \frac{N-Z}{A}) = \mathcal{E}_0(\rho_0, \frac{N-Z}{A} = 0) + \frac{\mathrm{d}^2\mathcal{E}}{\mathrm{d}\rho^2}\Big|_{\rho_0} (\rho-\rho_0)^2 + \dots$

$\rho = \rho_0 + K(\rho_0)\,(\rho-\rho_0)^2 + \dots$ justified for $\rho(\text{neutron star}) = 3\rho_0$??

Compressibility of nuclear matter $K(\rho) = 9\rho\frac{\mathrm{d}^2\mathcal{E}}{\mathrm{d}\rho^2} > 0$ for stable nuclear matter at density ρ.

Test dependence on $(\rho, N-Z)$ in neutron skin of heavy nuclei, collective excitations & **extrapolate**!

At ρ_0, $N = Z$: compressibility $K = k_F^2(\rho_0)\frac{\mathrm{d}^2\mathcal{E}}{\mathrm{d}\rho^2}\Big|_{\rho_0} = [210 \pm 10]\text{MeV}$. Wide agreement.

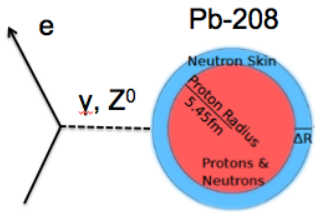
At ρ_0, pure neutron matter: $K \approx 600\text{MeV}$, error $\pm 100\text{MeV}$ or more.

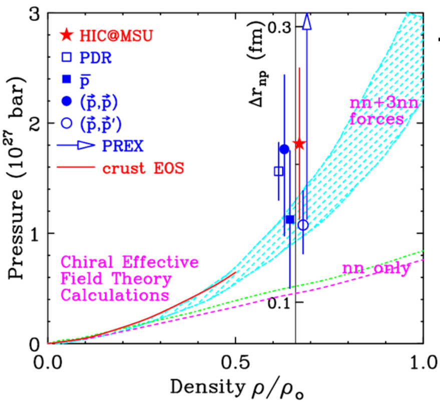
People disagree! Number here from [Vretenar/…PRC68 (2003) 024310]

Measure Compressibility & Neutron Skin? PREX at JLab & Co. . .

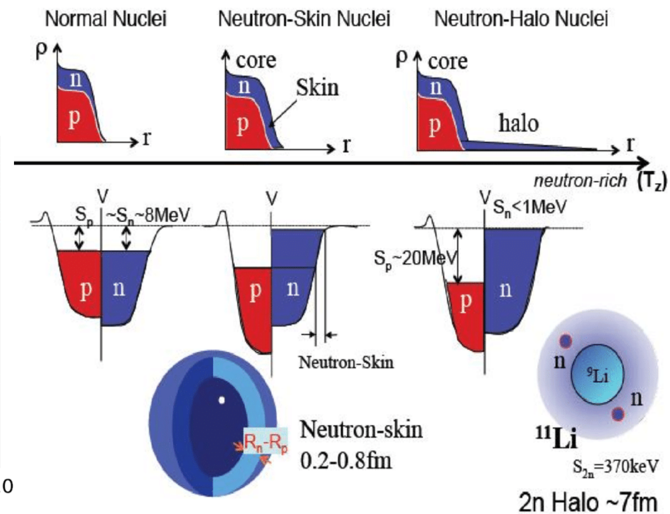
Problem: Neutrons have no charge $\Longrightarrow$ higher-order effect & weak interactions.

Inference Depends on Theory: skin only skin-deep, not "nuclear matter",. . .



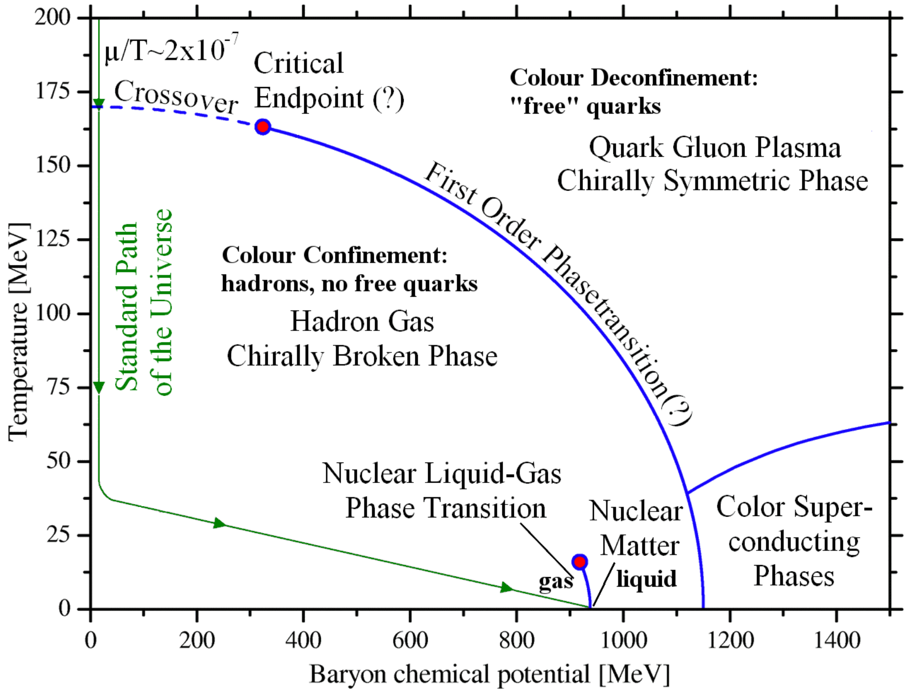


Experimental values of the ^{208}Pb neutron skin thickness (Δr_{np}), which is related to the neutron matter pressure at $\rho \approx 2/3\ \rho_0$, agree better with calculations that include 3-nucleon forces.



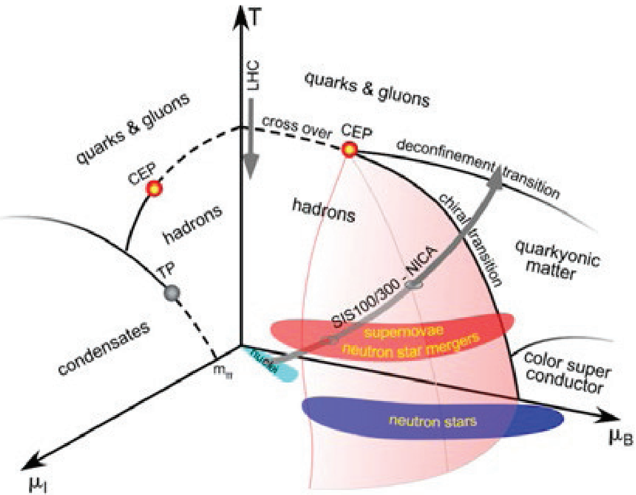
Preview: Nuclear Matter Phase Diagram for $N = Z$

When you compress nucleons, additional energy can be converted into new particles:
baryons ($\Lambda(1440)$,…) mesons (kaon,…), resonances/excitations ($\Delta(1232$,…), exotics…
$\Longrightarrow$ Influence on neutron-star radius,…! $\Longrightarrow$ Later.



Preview: Nuclear Matter Phase Diagram for $N \neq Z$

Need third axis with chemical potential $\mu_I = \mu_p - \mu_n$ for $Z - N$ to place neutron stars.



Box 3

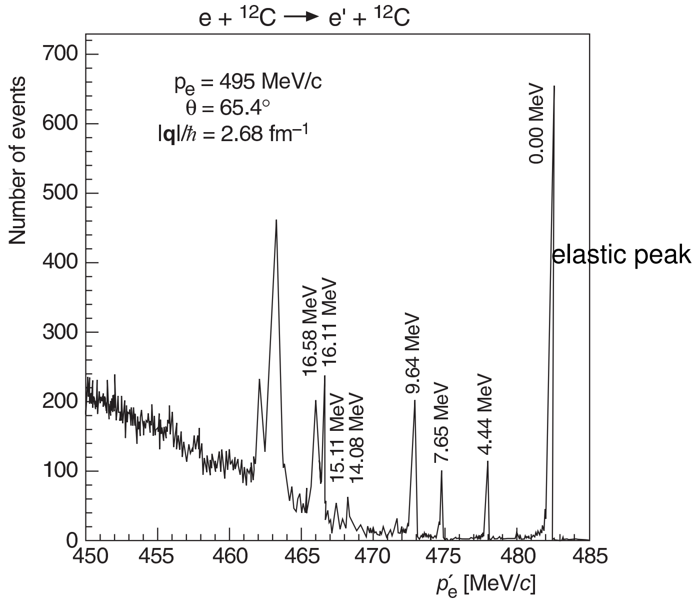
Features of the QCD Phase Diagram at Low Temperature and High Density

The 3-dimensional QCD phase diagram at high baryonic μ_B and moderate isospin μ_I densities has a rich and yet largely unexplored structure: a critical endpoint separates a smooth cross-over from a first order as well as a chiral phase transition at high baryon densities. New and exotic phases like quarkyonic matter or color superconducting phases might appear at baryonic high densities. At very high μ_B a superfluid color-flavor-locked phase is speculated on. Supernovae are formed at initial proton fractions ≈ 0.4 which reduce to ≈ 0.1 for cold neutron stars. Heavy-ion collisions at FAIR or NICA energies are expected to probe this region as well as the conjectured phase boundaries to quarkyonic or fully deconfined matter.

[NuPECC Long-Range Plan 2017 p. 89]

(f) Inelasticities: Excitations, Breakup, Knockout

SEMF does not explain nuclear level spectrum.



[PRSZR]

Fig. 5.9. Spectrum of electron scattering off ^{12}C. The sharp peaks correspond to elastic scattering and to the excitation of discrete energy levels in the ^{12}C nucleus by inelastic scattering. The excitation energy of the nucleus is given for each peak. The 495 MeV electrons were accelerated with the linear accelerator MAMI-B in Mainz and were detected using a high-resolution magnetic spectrometer (cf. Fig. 5.4) at a scattering angle of 65.4°. (*Courtesy of Th. Walcher and G. Rosner, Mainz*)

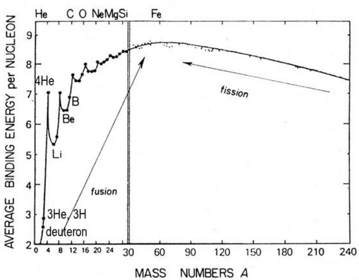
(g) Beyond the SEMF/Liquid Drop

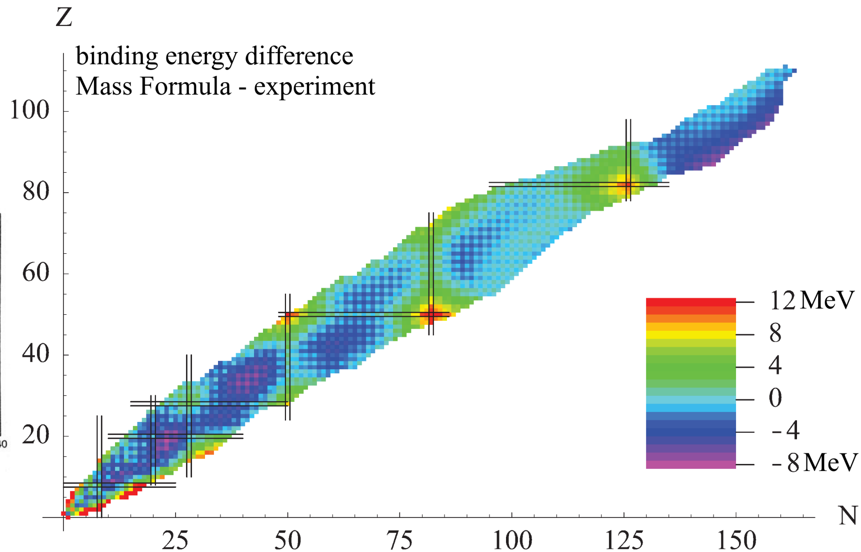
cursory look at [PRSZR 18, 19]

Difference Semi-Empirical Mass Formula SEMF – Experiment

Bethe-Weizsäcker: Semi-Empirical Mass Formula, good for qualitative arguments.

Magic numbers $2, 8, 20, 28, 50, 82, 126$ for Z or N more stable than SEMF $\Longrightarrow$ Shell-like structure?





Example of Single-Particle Models: 3 Minutes on the Shell Model

Single-Particle Models: Individual nucleon moves in average potential created by all other nucleons.

$\Longrightarrow$ Neglect feedback of motion onto potential. Saturation, short-range forces $\Longrightarrow V(r) \propto \rho(r)$

Light Nuclei: Gaußian profile; Heavy nuclei: Fermi/**Woods-Saxon potential** $V(r) = \dfrac{-V_0}{1 + \exp \frac{r-c}{a}}$

Full QM: Solve Schrödinger Equation

Analytically solvable models provide insight:

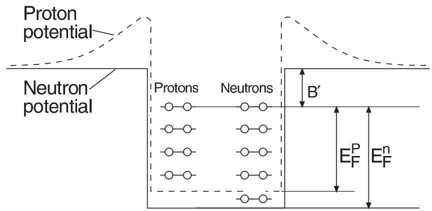
– **Fermi Gas/Liquid Model**: 3-dim. potential square-well with depth V_0.

– **3-dim Harm. Oscillator** $E_{\text{h.o.}} = (N_x + N_y + N_z + \frac{3}{2})\hbar\omega$; ang. mom. $l = N - 2(=\#\text{ wf nodes} - 1)$

Refinement Coulomb:

proton sees charges, neutron not.

$\Longrightarrow V_0^p > V_0^n$.



[PRSZR 18.1]

Refinement Spin-Orbit Coupling $V_{ls}(r)\vec{l}\cdot\vec{s}$: like (??) fine structure in H atom, where it is tiny $\mathcal{O}(\alpha^2)$.

Nucleon $\vec{s}\otimes\vec{l} = \vec{j} \Longrightarrow l \in \{j - \frac{1}{2}; j + \frac{1}{2}\} \Longrightarrow \vec{l}\cdot\vec{s} = \frac{1}{2}[(\vec{l}+\vec{s})^2 - \vec{l}^2 - \vec{s}^2] = \frac{1}{2}[j(j+1) - l(l+1) - \frac{3}{4}]$

$\Longrightarrow \Delta E_{ls} = (l + \frac{1}{2})\,\langle V_{ls}\rangle$ **Experiment:** $\langle V_{ls}\rangle \approx -20\text{MeV} < 0$ **huge**

(heavy & close constituents), opposite sign to H atom.

And, of course, many more refinements…

Example of Single-Particle Models: 3 Minutes on the Shell Model

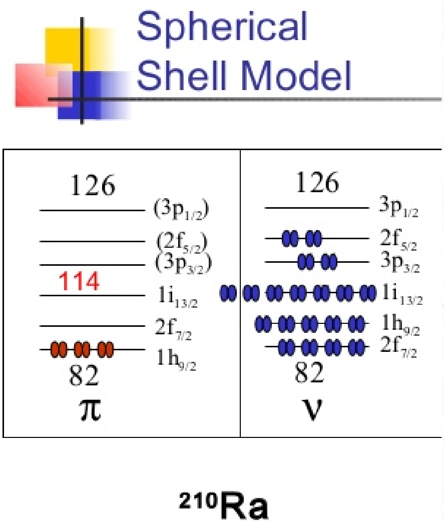
Each state with 2 protons & 2 neutrons (spin!); pairing $\Longrightarrow$ closed shells do not contribute.

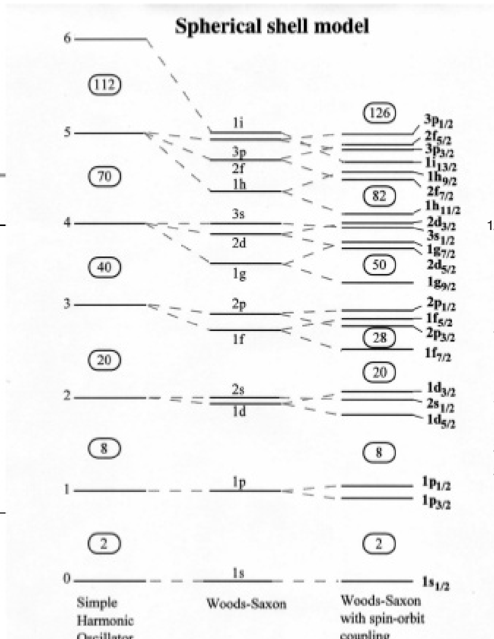
$\Longrightarrow$ **Gaps** at **magic numbers** $2, 8, 20, 28, 50, 82, 126$.

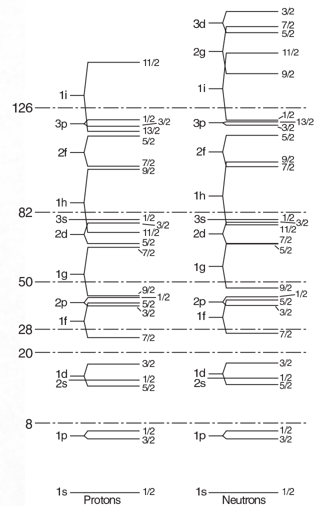
$\Longrightarrow$ Spin-orbit responsible for **gaps** at **magic numbers** $28, 50, 82, 126$.

[Maria Goeppert Mayer/Wigner/Jensen 1949 + developments: "Periodic table" of nuclei]

Very good *close to shell closure* ("valence nucleons"; incl. magnetic moments!), bad-ish off-closure.



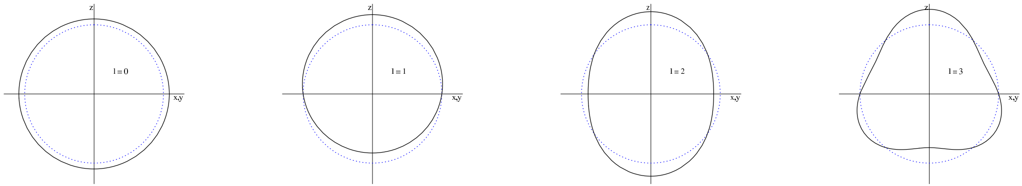




Liquid Drop Is Example of A Collective Model

Collective Model: **Nucleons loose individuality, form continuous fluid/gas.**

Example Collective Vibrations/Shape Oscillations: shape of nucleus deformed.



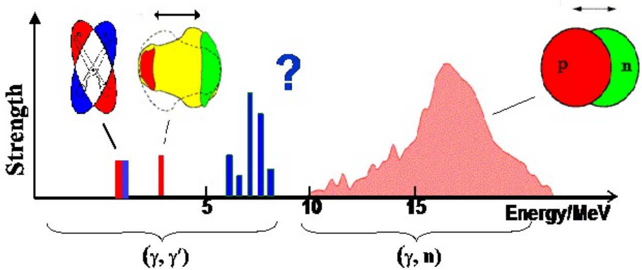
Example Compressibility of Nuclear Matter: “monopole mode” $J^{PC} = 0^{+-}$: radial oscillations.

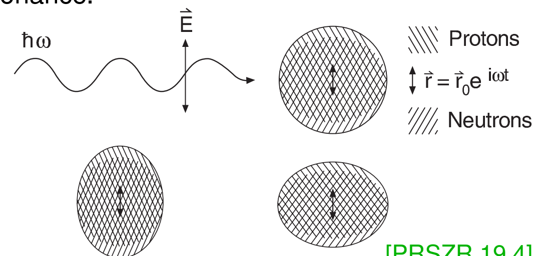
Experiment: excitation energy $\approx 80A^{-1/3}\text{MeV} \gg$ any other mode

$\Longrightarrow$ Nuclear matter pretty incompressible (except for interior of Neutron stars!).

Example Giant Electromagnetic Dipole Resonance: p & n oscillate against each other.

$\Longrightarrow$ Coherent elmag. excitation $\propto Z^2$; huge resonance.





[PRSZR 19.4]

Example Collective Rotations

Non-spherical nucleus rotates around non-symmetry axis, inertia I:

$$E_{\text{rot}} = \frac{\vec{J}^2}{2I} = \frac{J(J+1)}{2I} \quad \text{“rotation bands”}$$

$\Longrightarrow$ characteristic spacing

$$\Delta E \propto (2J+1).$$

Experiment: Inertia $I <$ rigid ellipsoid,

but $I >$ irrotational flow (superfluid)

$\Longrightarrow$ Nucleus like raw egg.

[PRSZR 18.14]

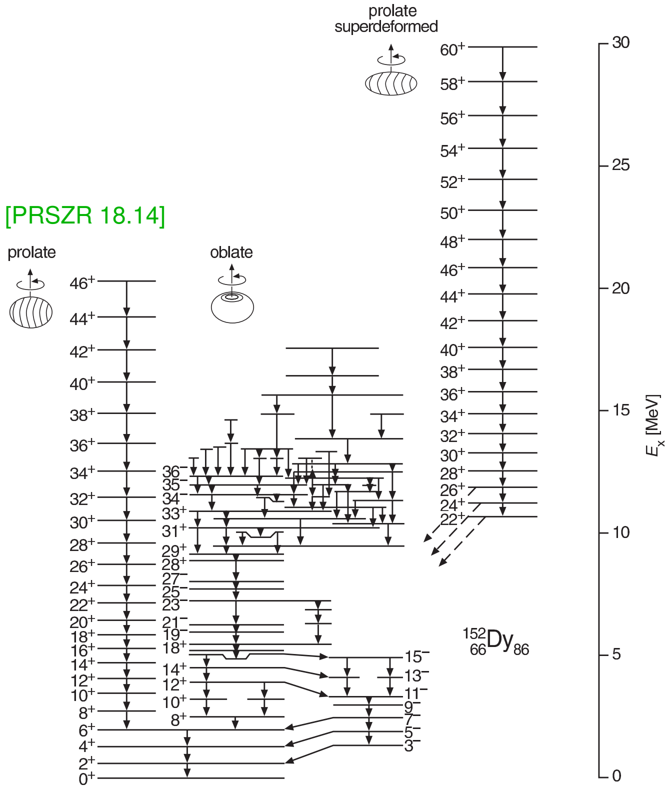


Fig. 18.14. Energy levels of ^{152}Dy [Sh90]. Although the low energy levels do not display typical rotation bands, these are seen in the higher excitations, which implies that the nucleus is then highly deformed.

Next: 2. Hadron Form Factors & Radii

Familiarise yourself with: [HM 8.2 (th); HG 6.5/6; Tho 7.5; Ann. Rev. Nucl. Part. Sci. 54 (2004) 217]