



THE GEORGE
WASHINGTON
UNIVERSITY
WASHINGTON, DC

H. W. Griebhammer

Institute for Nuclear Studies
The George Washington University
Spring 2023



III. Descriptions

5. (Electro-)Weak Interactions: The Glashow-Salam-Weinberg Theory

Or: A Theorist's Theory

References: [phenomenology: PRSZR 10, 11, 12, 18.6; Per 7.1-6 – theory: Ryd 8.3-5; CL 11, 12; Per 7, 8, 5.4;
most up-to-date: PDG 10-14 and reviews inside listings]



Leptonic Processes (Examples)

numbers from [PDG 2022]

Involve only leptons – rarest but cleanest \Rightarrow Use them to develop general theory!

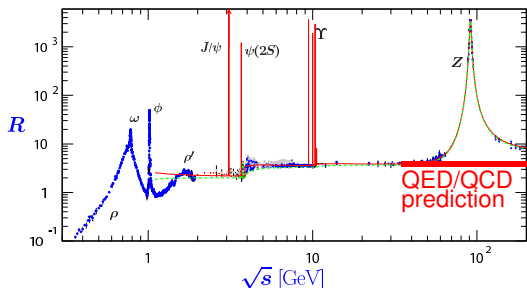
μ -Decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ $\tau \sim 10^{-6}$ s Both violate **individual** lepton conservation,

Charge Transfer: $e^- + \nu_\mu \rightarrow \mu^- + \nu_e$ but lepton-**family** number conserved:

$$\Rightarrow L_\mu(\mu^-) = L_\mu(\nu_\mu) = 1 = -L_\mu(\mu^+) = -L_\mu(\bar{\nu}_\mu) \text{ etc.}$$

In both, charge is transferred between leptons: **Charged-Current interaction (CC)**

The Z^0 resonance: wide, at $\sqrt{s} = 91$ GeV in $e^+e^- \rightarrow X$ decay 20% into $\nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$ pairs.



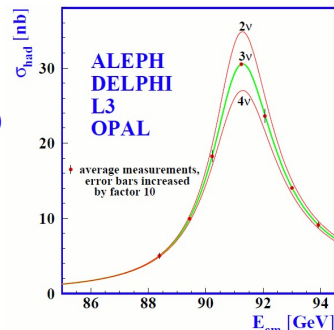
\Rightarrow Speculate weak process, mediated by $J^{PC} = 1^{--}$ boson:

Neutral-Current interaction (NC)

Determine ν rates indirectly:

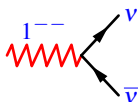
$$\Gamma_\nu = \underbrace{\Gamma_{\text{tot}}}_{\text{line shape}} - \underbrace{(\Gamma_{\text{had}} + \Gamma_{e\mu\tau})}_{\text{calorimeters}}$$

No decays like $\nu_e\bar{\nu}_\mu$ observed!



$$\Gamma[\rightarrow e^+e^-] : \Gamma[\rightarrow \mu^+\mu^-] : \Gamma[\rightarrow \tau^+\tau^-] = 1 : [1.0001 \pm 0.0024] : [1.0020 \pm 0.0032]$$

\Rightarrow Weak interaction universal for both neutrinos and charged leptons.



GSW: $\Gamma[\text{invisible}] = [499.0 \pm 1.5] \text{ MeV}$. HW 5.5: LO decay $\Gamma = \frac{g^2 M_W}{12\pi}$, $g \rightarrow \dots$ GSW theory

Compare to $\Gamma_\nu^{\text{exp}} \Rightarrow [2.92 \pm 0.05] \nu$ species with $M_\nu \ll 90$ GeV

Semi-Leptonic Processes (Examples)

Involve leptons and hadrons – most common, oldest seen.



Uranium decay

unspecific: **Henri Bequerel 26 February 1896**

Neutron decay

$$n(udd) \rightarrow p(uud) + e^- + \bar{\nu}_e \quad \tau = [878.4 \pm 0.5]s \quad [\text{PDG 2022}]$$

i.e. $d \rightarrow ue^- \bar{\nu}_e \implies$ **Charged Current Exchange: CC**

π decay, e.g.

$$\pi^+(u\bar{d}) \rightarrow \mu^+ + \nu_\mu, e^+ + \nu_e, \text{ i.e. quark process similar to proton} \quad \text{CC}$$

K decay, e.g.

$$K^+(u\bar{s}) \rightarrow \mu^+ + \nu_\mu, e^+ + \nu_e, \text{ i.e. } u\bar{s} \rightarrow (s\bar{s} \text{ or } u\bar{u}) \rightarrow \dots \quad \text{CC}$$

Solar fusion

$$p + p \rightarrow {}^2\text{H} + e^+ + \nu_e \quad \text{kind of important...} \quad \text{CC}$$

Nuclear β decay

$$\text{e.g. } {}^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \bar{\nu}_e \quad \text{Wu 1957: } P \text{ violated} \quad \text{CC}$$

Nuclear e^- -capture

$$\text{e.g. } e^- + {}^{152}\text{Eu}(J=0) \rightarrow {}^{152}\text{Sm}(J=0) + \gamma + \nu_e \quad \text{CC}$$

Goldhaber 1958: ν helicity measurement

All above mutate quark flavours: individual quark-number violated.

$$\nu_l + A \rightarrow \nu_l + X$$

No charged lepton in final state $\implies Z^0!$

First Neutral-Current (NC) event [CERN 1973; GSW prediction]

NC

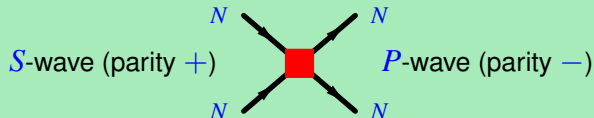
Hadronic Processes (Examples)

Involve only hadrons – window to QCD.

K decay $K^0(d\bar{s}) \rightarrow \pi^+(u\bar{d}) + \pi^-(\bar{u}d)$, i.e. $\bar{s} \rightarrow \bar{d} + u\bar{u}$ CC

$\Lambda(1405)$ decay $\Lambda^0(uds) \rightarrow p(uud) + \pi^-(\bar{u}d)$ $\tau \sim 10^{-10}$ s CC

Research Frontier: Hadronic flavour-conserving parity-violation (HFCPV), e.g. $pp \rightarrow pp$



One of the least-explored sectors of the Standard Model:

GW theory: hgrie

- What is the weak part of the nuclear force? (US, EU Long Range Plans)

- Z^0 (NC) as Inside-Out Probe of non-perturbative QCD: qq correlations at $\frac{1}{M_W} \sim 0.002$ fm

What we find – and what not (Examples)

Neutral & Charged Current Exchanges with $J^{PC} = 1^{--}$, **like for photon**:

Produced as resonances in annihilations and other processes:

$$e^+e^-(\sqrt{s} = 90\text{GeV}) \rightarrow Z^0, e^+e^-(\sqrt{s} = 160\text{GeV}) \rightarrow W^+W^-;$$

and in NN or $N\bar{N}$ collisions also resonances from $u\bar{u} \rightarrow Z^0, u\bar{d} \rightarrow W^+$.

⇒ **Try gauge theory of gauge bosons with charges $\pm 1, 0$?**

Not/Rarely Seen

Frequently Seen

Interpretation

$$\bar{\nu}_e + n \not\rightarrow e^- + p$$

$$\nu_e + n \rightarrow e^- + p$$

neutrino is not anti-neutrino, $L_e(\nu_e) = -L_e(\bar{\nu}_e)$

$$\bar{\nu}_\mu + p \not\rightarrow e^+ + n$$

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

e -neutrino is not μ -neutrino, but...

$$\nu_\mu + A \not\rightarrow e^- + X$$

$$\nu_\mu + A \rightarrow \mu^- + X$$

no interactions across lepton families

⇒ **Natural grouping into lepton families:** $\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

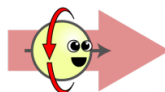
(b) Weak Interactions Violate Parity

Reminder Fermion Helicity & Chirality

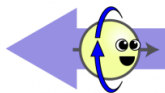
[QFT and TCP chapters]

Helicity $h = \frac{\vec{\sigma} \cdot \vec{p}}{E}$:

spin component longitudinal to \vec{p}



parallel: **right-handed** $h = +1$



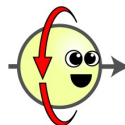
anti-parallel: **left-handed** $h = -1$

For $m = 0$, identical to **chirality**: eigenvalues of spinors with respect to γ_5 :

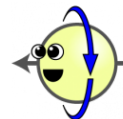
$$\gamma_5 \varphi_{RL} = \pm \varphi_{RL}$$

Projectors: $P_{RL} := \frac{1}{2}(1 \pm \gamma_5)$, i.e. $P_{RL}\varphi = \varphi_{RL}$, $P_{RL}^2 = P_{RL}$, $P_{RL}P_{LR} = 0$, $P_R + P_L = 1$

Parity transformation: $\vec{\sigma}$ axial, \vec{p} polar $\implies Ph_{\pm} = h_{\mp}$



parity
 \implies



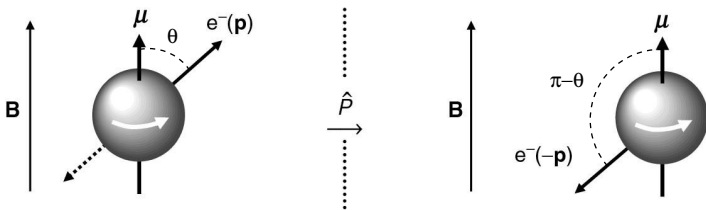
Recall Gauge Theory

Lagrangian in chiral basis: $(\varphi_R^\dagger, \varphi_L^\dagger) \begin{pmatrix} E - gA_0 + \vec{\sigma} \cdot (\vec{p} + g\vec{A}) & m \\ m & E + gA_0 - \vec{\sigma} \cdot (\vec{p} - g\vec{A}) \end{pmatrix} \begin{pmatrix} \varphi_R \\ \varphi_L \end{pmatrix}$

\implies **Gauge field does not mix chiralities; only mass term does:** $\propto (1 - \beta) = 1 - \frac{|\vec{p}|}{E}$.

Electron Helicity from Nuclear β Decay (CC Event)

First: Wu 1957 (prompted by theorists Lee/Yang 1956) ${}^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \bar{\nu}_e$



Reflection on plane perpendicular to $\vec{\mu}$: $\hat{P}\vec{p} \rightarrow -\vec{p}$, $\hat{P}\vec{\mu} \rightarrow \vec{\mu}$

Result: Intensity $I(\theta) \neq I(\pi - \theta)$, and emission of e^- more likely **against ${}^{60}\text{Co}$ spin**, matches dependence on initial e^- -polarisation P :

$$I(\theta) = 1 + P \frac{\vec{\sigma}_e \cdot \vec{p}_e}{E_e} = 1 + P \beta_e \cos \theta$$

and data compatible with $P = -1$.

\Rightarrow **Parity violated, electron emitted with $h_e = -1$, $m_e \neq 0$ explains spin-flip observed in detector.**

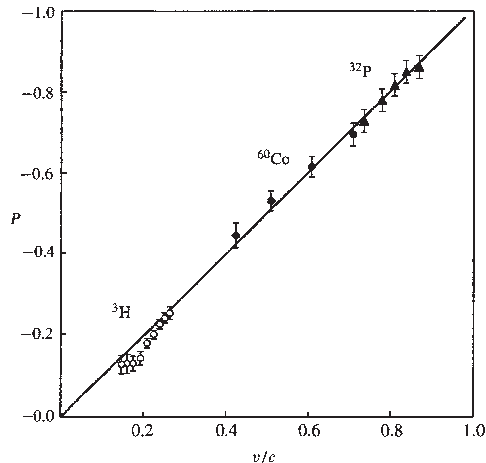
Similar for $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$: $P(e^+) = +1$.

Both confirmed in cornucopia of systems.

$\vec{B} \cong \vec{e}_z$ defines quantisation axis for ${}^{60}\text{Co}$ spin $\vec{\mu}$ and e^- spin $\vec{\sigma}_e$.

Expectation if parity conserved:

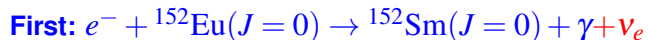
e^- emission uniform $I(\theta) = I(\pi - \theta)$.



[Per 7.6 after Koks/van Klinken 1976]

Fig. 7.6. The polarisation P of electrons emitted in nuclear β -decay, plotted as a function of electron velocity. The results demonstrate that $P = -v/c$, as in (7.16). After Koks and Van Klinken (1976).

Neutrino Helicity from Nuclear Capture (CC event) cf. [PRSZR 18.6, Per 7.6]



Goldhaber 1958

J_z conservation: photon spin ($J=1$) **parallel** to electron spin ($J=\frac{1}{2}$), **antiparallel** to ν spin ($J=\frac{1}{2}$).

\Rightarrow **Detect photon spin to know ν helicity** (mag. quantum $m_e = m_\gamma + m_\nu$).

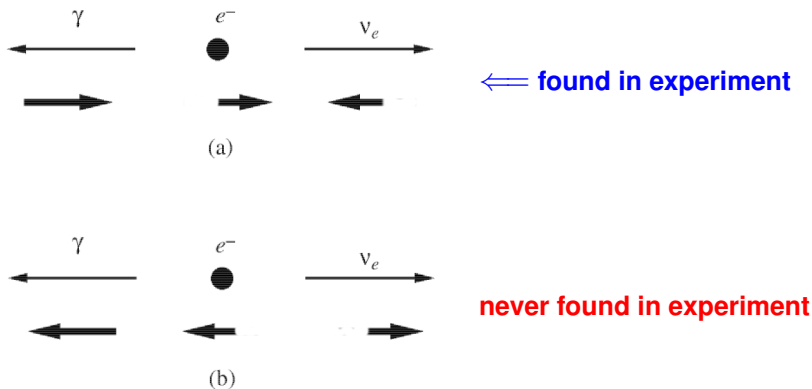


Figure 6.6 Possible helicities of the photon and neutrinos emitted in the reaction $e^- + {}^{152}\text{Eu}(J=0) \rightarrow {}^{152}\text{Sm}(J=0) + \nu_e + \gamma$ for those events in which they are emitted in opposite directions. Experiment selects configuration (a) [Mar]

\Rightarrow **All evidence suggests: only e^-_L , ν_L and e^+_R , $\bar{\nu}_R$ interact weakly in CC events: Maximal Parity Violation**

(c) Philosophy of the Glashow-Salam-Weinberg Model

Universality of weak interactions: simplifies inelegance of 1 coupling per $q\ell\nu$ interaction.

“Theorist’s Theory”: As simple as possible, as flexible as necessary, and compulsory unless forbidden.

First construct sector for one lepton family (e, ν_e):

(1) Start from massless fermions: $\Psi_{L/R}$ are eigenstates of γ_5 – generate masses later.

(2) Postulate: only l_L, \bar{l}_R couple to W^\pm , but not l_R, \bar{l}_L .

(3) \implies Right-handed leptons
Left-handed anti-leptons emerge from W^\pm process only after conversion by mass term m_l .

(4) Charged & neutral weak currents mediated by $J^{PC} = 1^{--}$ gauge bosons W^\pm, Z^0 .

Non-Abelian gauge principle already successful in QCD (QED).

(5) Photon and Z^0 both 1^{--} gauge bosons with no elmag. charge. \implies Can mix (Swiss Law).

(6) We still need a massless photon: need a $U(1)_Y$ group somewhere (weak hypercharge).

(7) Lepton-family number conservation:

$$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \bar{l}_R = \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix} \text{ weak (iso-)doublets: } SU(2)_L \text{ and } U(1)_Y \text{ act on them.}$$

$e_R, \nu_{eR}, \bar{e}_L, \bar{\nu}_{eL}$ weak (iso-)singlets: $SU(2)_L$ does *not* act on them, but $U(1)_Y$ does.

(8) Same for the other lepton families $\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \nu_{\mu R}, \mu_R, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \nu_{\tau R}, \tau_R$

(c) Philosophy of the Glashow-Salam-Weinberg Model

Universality of weak interactions: simplifies inelegance of 1 coupling per $q\ell\nu$ interaction.

“Theorist’s Theory”: As simple as possible, as flexible as necessary, and compulsory unless forbidden.

First construct sector for one lepton family (e, ν_e):

(1) Start from massless fermions: $\Psi_{L/R}$ are eigenstates of γ_5 – generate masses later.

(2) Postulate: only l_L, \bar{l}_R couple to W^\pm , but not l_R, \bar{l}_L .

(3) \implies Right-handed leptons emerge from W^\pm process only after conversion by mass term m_l .
Left-handed anti-leptons

(4) Charged & neutral weak currents mediated by $J^{PC} = 1^{--}$ gauge bosons W^\pm, Z^0 .

Non-Abelian gauge principle already successful in QCD (QED).

(5) Photon and Z^0 both 1^{--} gauge bosons with no elmag. charge. \implies Can mix (Swiss Law).

(6) We still need a massless photon: need a $U(1)_Y$ group somewhere (weak hypercharge).

(7) Lepton-family number conservation:

$$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \bar{l}_R = \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix} \text{ weak (iso-)doublets: } SU(2)_L \text{ and } U(1)_Y \text{ act on them.}$$

$e_R, \nu_{eR}, \bar{e}_L, \bar{\nu}_{eL}$ weak (iso-)singlets: $SU(2)_L$ does *not* act on them, but $U(1)_Y$ does.

(8) Same for the other lepton families $\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \nu_{\mu R}, \mu_R, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \nu_{\tau R}, \tau_R$

(c) Philosophy of the Glashow-Salam-Weinberg Model

Universality of weak interactions: simplifies inelegance of 1 coupling per $q\ell\nu$ interaction.

“Theorist’s Theory”: As simple as possible, as flexible as necessary, and compulsory unless forbidden.

First construct sector for one lepton family (e, ν_e):

(1) Start from massless fermions: $\Psi_{L/R}$ are eigenstates of γ_5 – generate masses later.

(2) Postulate: only l_L, \bar{l}_R couple to W^\pm , but not l_R, \bar{l}_L .

(3) \implies **Right-handed leptons** emerge from W^\pm process only after conversion by mass term m_l .
Left-handed anti-leptons

(4) Charged & neutral weak currents mediated by $J^{PC} = 1^{--}$ gauge bosons W^\pm, Z^0 .

Non-Abelian gauge principle already successful in QCD (QED).

(5) Photon and Z^0 both 1^{--} gauge bosons with no elmag. charge. \implies Can mix (Swiss Law).

(6) We still need a massless photon: need a $U(1)_Y$ group somewhere (weak hypercharge).

(7) Lepton-family number conservation:

$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \bar{l}_R = \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix}$ weak (iso-)doublets: $SU(2)_L$ and $U(1)_Y$ act on them.

$e_R, \nu_{eR}, \bar{e}_L, \bar{\nu}_{eL}$ weak (iso-)singlets: $SU(2)_L$ does *not* act on them, but $U(1)_Y$ does.

(8) Same for the other lepton families $\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \nu_{\mu R}, \mu_R, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \nu_{\tau R}, \tau_R$

(c) Philosophy of the Glashow-Salam-Weinberg Model

Universality of weak interactions: simplifies inelegance of 1 coupling per $q\ell\nu$ interaction.

“Theorist’s Theory”: As simple as possible, as flexible as necessary, and compulsory unless forbidden.

First construct sector for one lepton family (e, ν_e):

(1) Start from massless fermions: $\Psi_{L/R}$ are eigenstates of γ_5 – generate masses later.

(2) Postulate: only l_L, \bar{l}_R couple to W^\pm , but not l_R, \bar{l}_L .

(3) \implies Right-handed leptons emerge from W^\pm process only after conversion by mass term m_l .
Left-handed anti-leptons

(4) Charged & neutral weak currents mediated by $J^{PC} = 1^{--}$ gauge bosons W^\pm, Z^0 .

Non-Abelian gauge principle already successful in QCD (QED).

(5) Photon and Z^0 both 1^{--} gauge bosons with no elmag. charge. \implies Can mix (Swiss Law).

(6) We still need a massless photon: need a $U(1)_Y$ group somewhere (weak hypercharge).

(7) Lepton-family number conservation:

$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \bar{l}_R = \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix}$ weak (iso-)doublets: $SU(2)_L$ and $U(1)_Y$ act on them.

$e_R, \nu_{eR}, \bar{e}_L, \bar{\nu}_{eL}$ weak (iso-)singlets: $SU(2)_L$ does *not* act on them, but $U(1)_Y$ does.

(8) Same for the other lepton families $\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \nu_{\mu R}, \mu_R, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \nu_{\tau R}, \tau_R$

(c) Philosophy of the Glashow-Salam-Weinberg Model

Universality of weak interactions: simplifies inelegance of 1 coupling per $q\ell\nu$ interaction.

“Theorist’s Theory”: As simple as possible, as flexible as necessary, and compulsory unless forbidden.

First construct sector for one lepton family (e, ν_e):

(1) Start from massless fermions: $\Psi_{L/R}$ are eigenstates of γ_5 – generate masses later.

(2) Postulate: only l_L, \bar{l}_R couple to W^\pm , but not l_R, \bar{l}_L .

(3) \implies **Right-handed leptons** emerge from W^\pm process only after conversion by mass term m_l .
Left-handed anti-leptons

(4) Charged & neutral weak currents mediated by $J^{PC} = 1^{--}$ gauge bosons W^\pm, Z^0 .

Non-Abelian gauge principle already successful in QCD (QED).

(5) Photon and Z^0 both 1^{--} gauge bosons with no elmag. charge. \implies Can mix (Swiss Law).

(6) We still need a massless photon: need a $U(1)_Y$ group somewhere (weak hypercharge).

(7) Lepton-family number conservation:

$$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \bar{l}_R = \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix} \text{ weak (iso-)doublets: } SU(2)_L \text{ and } U(1)_Y \text{ act on them.}$$

$e_R, \nu_{eR}, \bar{e}_L, \bar{\nu}_{eL}$ weak (iso-)singlets: $SU(2)_L$ does *not* act on them, but $U(1)_Y$ does.

(8) Same for the other lepton families $\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \nu_{\mu R}, \mu_R, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \nu_{\tau R}, \tau_R$

(c) Philosophy of the Glashow-Salam-Weinberg Model

Universality of weak interactions: simplifies inelegance of 1 coupling per $q\ell\nu$ interaction.

“Theorist’s Theory”: As simple as possible, as flexible as necessary, and compulsory unless forbidden.

First construct sector for one lepton family (e, ν_e):

(1) Start from massless fermions: $\Psi_{L/R}$ are eigenstates of γ_5 – generate masses later.

(2) Postulate: only l_L, \bar{l}_R couple to W^\pm , but not l_R, \bar{l}_L .

(3) \implies Right-handed leptons emerge from W^\pm process only after conversion by mass term m_l .
Left-handed anti-leptons

(4) Charged & neutral weak currents mediated by $J^{PC} = 1^{--}$ gauge bosons W^\pm, Z^0 .

Non-Abelian gauge principle already successful in QCD (QED).

(5) Photon and Z^0 both 1^{--} gauge bosons with no elmag. charge. \implies Can mix (Swiss Law).

(6) We still need a massless photon: need a $U(1)_Y$ group somewhere (weak hypercharge).

(7) Lepton-family number conservation:

$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \bar{l}_R = \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix}$ weak (iso-)doublets: $SU(2)_L$ and $U(1)_Y$ act on them.

$e_R, \nu_{eR}, \bar{e}_L, \bar{\nu}_{eL}$ weak (iso-)singlets: $SU(2)_L$ does *not* act on them, but $U(1)_Y$ does.

(8) Same for the other lepton families $\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \nu_{\mu R}, \mu_R, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \nu_{\tau R}, \tau_R$

(c) Philosophy of the Glashow-Salam-Weinberg Model

Universality of weak interactions: simplifies inelegance of 1 coupling per $q\bar{l}\nu$ interaction.

“Theorist’s Theory”: As simple as possible, as flexible as necessary, and compulsory unless forbidden.

First construct sector for one lepton family (e, ν_e):

- (1) Start from massless fermions: $\Psi_{L/R}$ are eigenstates of γ_5 – generate masses later.
- (2) Postulate: only l_L, \bar{l}_R couple to W^\pm , but not l_R, \bar{l}_L .
- (3) \implies

Right-handed leptons

Left-handed anti-leptons

 emerge from W^\pm process only after conversion by mass term m_l .
- (4) Charged & neutral weak currents mediated by $J^{PC} = 1^{--}$ gauge bosons W^\pm, Z^0 .
Non-Abelian gauge principle already successful in QCD (QED).
- (5) Photon and Z^0 both 1^{--} gauge bosons with no elmag. charge. \implies Can mix (Swiss Law).
- (6) We still need a massless photon: need a $U(1)_Y$ group somewhere (weak hypercharge).
- (7) Lepton-family number conservation:

$$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \bar{l}_R = \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix} \text{ weak (iso-)doublets: } SU(2)_L \text{ and } U(1)_Y \text{ act on them.}$$

$$e_R, \nu_{eR}, \bar{e}_L, \bar{\nu}_{eL} \text{ weak (iso-)singlets: } SU(2)_L \text{ does not act on them, but } U(1)_Y \text{ does.}$$

- (8) Same for the other lepton families $\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \nu_{\mu R}, \mu_R, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \nu_{\tau R}, \tau_R$

(c) Philosophy of the Glashow-Salam-Weinberg Model

Universality of weak interactions: simplifies inelegance of 1 coupling per $q\ell\nu$ interaction.

“Theorist’s Theory”: As simple as possible, as flexible as necessary, and compulsory unless forbidden.

First construct sector for one lepton family (e, ν_e):

(1) Start from massless fermions: $\Psi_{L/R}$ are eigenstates of γ_5 – generate masses later.

(2) Postulate: only l_L, \bar{l}_R couple to W^\pm , but not l_R, \bar{l}_L .

(3) \implies Right-handed leptons emerge from W^\pm process only after conversion by mass term m_l .
Left-handed anti-leptons

(4) Charged & neutral weak currents mediated by $J^{PC} = 1^{--}$ gauge bosons W^\pm, Z^0 .

Non-Abelian gauge principle already successful in QCD (QED).

(5) Photon and Z^0 both 1^{--} gauge bosons with no elmag. charge. \implies Can mix (Swiss Law).

(6) We still need a massless photon: need a $U(1)_Y$ group somewhere (weak hypercharge).

(7) Lepton-family number conservation:

$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \bar{l}_R = \begin{pmatrix} \bar{\nu}_{eR} \\ \bar{e}_R \end{pmatrix}$ weak (iso-)doublets: $SU(2)_L$ and $U(1)_Y$ act on them.

$e_R, \nu_{eR}, \bar{e}_L, \bar{\nu}_{eL}$ weak (iso-)singlets: $SU(2)_L$ does *not* act on them, but $U(1)_Y$ does.

(8) Same for the other lepton families $\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \nu_{\mu R}, \mu_R, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \nu_{\tau R}, \tau_R$

(d) GSW for One Lepton Family

(e) Dynamical Gauge Boson Mass Generation

Nobel 2013

The Higgs-Kibble-Englert Mechanism: A $U(1)$ Example



See Landau-Ginzburg Theory of Superconductivity

A Sketch of Dynamical Mass Generation in GSW

We want 3 massive and 1 massless vector fields, and “true” Higgs field ϕ not to couple to photon.

⇒ Choose complex Higgs doublet $\Phi(x)$, use $SU_L(2)$ gauge trafo to “Unitary Gauge”:

$$U(x)\Phi(x) = \begin{pmatrix} 0 \\ a + \frac{\varphi(x)}{\sqrt{2}} \end{pmatrix} \text{ with real (uncharged) scalar } \varphi(x); \quad \text{cf. weak anti-doublet } \begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}$$

One Can Show: can always be done: like rotating spin into z direction.

Question: Why is Higgs Vacuum Expectation Value (VEV) $a \neq 0$? — Answer: We do not know.

A Sketch of Dynamical Mass Generation in GSW

We want 3 massive and 1 massless vector fields, and “true” Higgs field ϕ not to couple to photon.

⇒ Choose complex Higgs doublet $\Phi(x)$, use $SU_L(2)$ gauge trafo to “Unitary Gauge”:

$$U(x)\Phi(x) = \begin{pmatrix} 0 \\ a + \frac{\varphi(x)}{\sqrt{2}} \end{pmatrix} \text{ with real (uncharged) scalar } \varphi(x); \quad \text{cf. weak anti-doublet } \begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}$$

One Can Show: can always be done: like rotating spin into z direction.

Question: Why is Higgs Vacuum Expectation Value (VEV) $a \neq 0$? — Answer: We do not know.

Determine weak hypercharge such that ϕ neutral: $0 \stackrel{!}{=} Q = T_3 + \frac{Y_\phi}{2} = -\frac{1}{2} + \frac{Y_\phi}{2} \implies Y_\phi = +1$

$$\implies D_\mu \Phi = \left[\partial_\mu \mathbb{1} - \frac{i}{2} \begin{pmatrix} \underbrace{g W_\mu^{(3)} + g' B_\mu}_{=\sqrt{g^2+g'^2} A_\mu \text{ photon}} & g\sqrt{2} W_\mu^+ \\ g\sqrt{2} W_\mu^- & \underbrace{-g W_\mu^{(3)} + g' B_\mu}_{=-\sqrt{g^2+g'^2} Z_\mu} \end{pmatrix} \right] \begin{pmatrix} 0 \\ a + \frac{\varphi(x)}{\sqrt{2}} \end{pmatrix}$$

Multiply out $(D_\mu \Phi)^\dagger (D^\mu \Phi)$: – massless photon A_μ

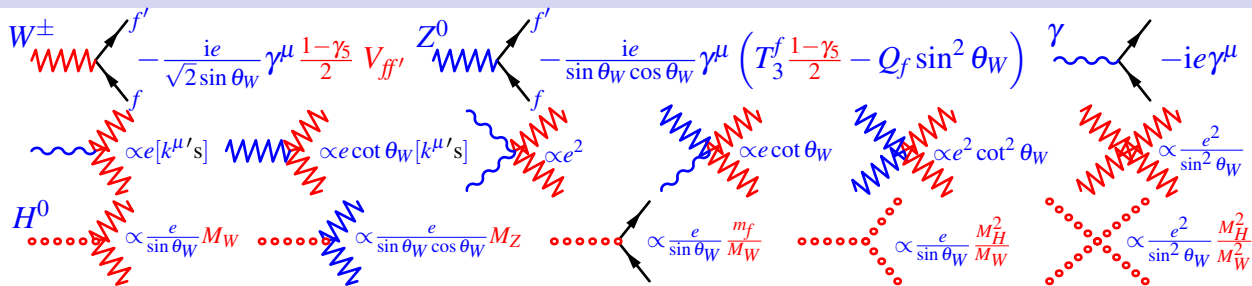
$$\text{– masses } M_W^2 = \frac{g^2 a^2}{2}, M_Z^2 = \frac{(g^2 + g'^2) a^2}{2} \implies \mu \text{ } \underbrace{\text{W}} \text{ } \nu : \frac{-i}{q^2 - M_{W,Z}^2} \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_{W,Z}^2} \right]$$

$$\text{with } \frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + g'^2} = \cos^2 \theta_W \text{ at “tree level” (before quantum corrections).}$$

(Tree-Level) Interactions and Experimental Numbers

[PDG 2022]

[AH II.Q.2.3]



W mass $M_W = [80.377 \pm 0.012]\text{GeV}$

Z mass $M_Z = [91.1876 \pm 0.0021]\text{GeV}$

Higgs mass $m_H = [125.25 \pm 0.17]\text{GeV}$

Weinberg mixing angle $\sin^2\theta_W(M_Z^2) = 0.23121(4)$ ($\theta_W(M_Z^2) \approx 29^\circ$)

Fermi coupling $G_F = 1.1663788(6) \times 10^{-5}\text{GeV}^{-2}$

Cabbibo angle ($= |V_{us}|/|V_{ud}|$) $\sin\theta_C = 0.2265(5)$ ($\theta_C = 13.091(27)^\circ$)

$$e \approx g \sin\theta_W \implies \frac{g^2}{4\pi} = \frac{\alpha}{\sin^2\theta_W} \approx \frac{1}{30} \gg \frac{1}{137} = \alpha \text{ "Weak Coupling not Weak".}$$

Higgs VEV $a \approx \sqrt{2}gM_W \approx 71\text{GeV}$

Higgs curvature $\lambda \approx \frac{m_H^2}{4a^2} \approx 1.5$ large: narrow valley (wide one would give large corrections).

Has most Nobels: Yang/Lee 1957 (th: P violation), Glashow/Slam/Weinberg 1979 (th: GSW), Cronin 1980 (ex: CP violation), Rubbia/Meer 1984 (ex: W, Z), Ledermann/Schwartz/Steinberger 1988 (ex: ν_μ), Perl 1995 (ex: τ), Reines 1995 (ex: ν), 't Hooft/Veltman 1999 (th: QFT of GSW), Davis/Koshiba 2002 (ex: cosmic ν), Kobayashi/Maskawa 2008 (th: CKM), Englert/Higgs 2013 (th: Higgs), Kajita/McDonald 2015 (ex: m_ν).

A Loose End: Fermion Masses by Yukawa Mechanism

So far, no fermion masses: helicity = chirality.

Since Higgs was so good at giving mass to W and Z , let it also generate m_f :

Yukawa Coupling (cf. πN): $\mathcal{L}_{\text{mass}} = \sum_{\text{all massive fermions } f} g_f [\bar{f}_L \Phi f_R + \bar{f}_R \Phi^\dagger f_L]$ couples L and R chirality

Use $\Phi = \begin{pmatrix} 0 \\ a + \frac{\varphi(x)}{\sqrt{2}} \end{pmatrix} \rightarrow \sum_f \underbrace{a g_f}_{= m_f \text{ fermion mass}} [\bar{f}_L f_R + \bar{f}_R f_L] + \text{fermion-Higgs interactions which increase with } m_f$

Economic but not elegant: one coupling per massive fermion $\implies 9$ (12) parameters.

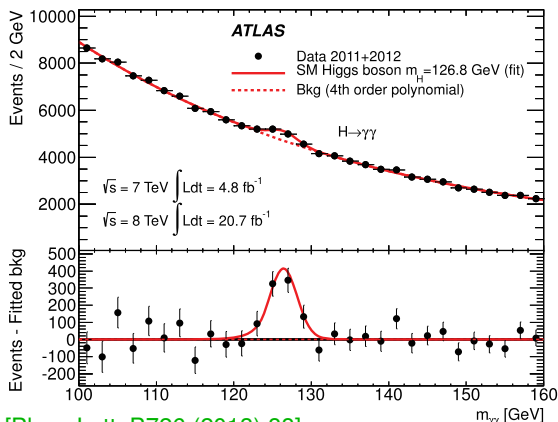
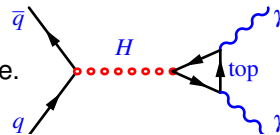
**Higgs does *not* explain nucleon masses $M_N \approx 940\text{MeV} \gg m_{u,d} \approx 4\text{MeV}$:
Vast majority of hadron mass (and therefore of visible-universe mass)
comes from **QCD**, not from Higgs (contrary to Particle Physicist Propaganda).**

Discovery 2012 at CERN's LHC ($p\bar{p}$ collider): ATLAS & CMS Collaborations.

Discovery channel $q\bar{q} \rightarrow H \rightarrow \gamma\gamma$:

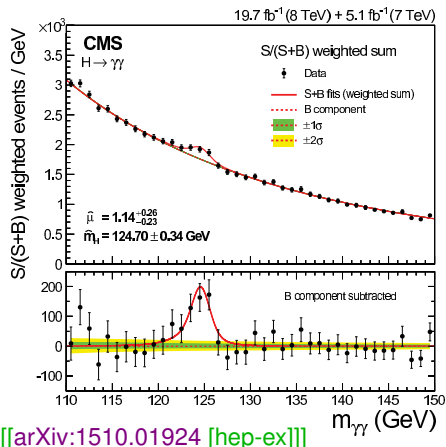
branching ratio **0.2%**, but very clean signature.

Via "top loop" since tH coupling $\propto m_t$ large.



[Phys. Lett. B726 (2013) 88]

Fig. 2. Invariant mass distribution of diphoton candidates after all selections of the inclusive analysis for the combined 7 TeV and 8 TeV data. The result of a fit to the data with the sum of a SM Higgs boson signal (with $m_H = 126.8$ GeV and free signal strength) and background is superimposed. The residuals of the data with respect to the fitted background are displayed in the lower panel.



[[arXiv:1510.01924 [hep-ex]]]

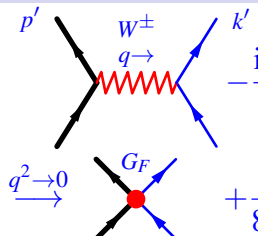
Figure 1: The $m_{\gamma\gamma}$ distribution as weighted sum of all categories [13]. S and B are the number of signal and background events, respectively.

By now, many other channels seen as well. – All results consistent with GSW/Standard Model.

(f) The (Low-Energy) EFT of GSW: Fermi's V-A Theory

$$\frac{g^2}{4\pi} \approx \frac{1}{30} \gg \alpha \approx \frac{1}{137}$$

⇒ Why “weak”?



$$- \frac{ig^2}{2} [\bar{U}_L(p') \gamma^\mu U_L(p)] \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} [\bar{u}_L(k') \gamma^\nu u_L(k)]$$

$$+ \frac{g^2}{8M_W^2} \underbrace{[\bar{U} \gamma_\mu (1 - \gamma_5) U]}_{=: J_\mu^{\text{weak}}} \underbrace{[\bar{u} \gamma^\mu (1 - \gamma_5) u]}_{=: 2\bar{u}_L \gamma^\mu U_L} + \mathcal{O}\left(\frac{q^2}{M_W^2}\right)$$

⇒ For momentum transfers $q^2 \ll M_W^2$, see **point-like coupling between Axial Currents** with

Fermi Constant $G_F = \frac{\sqrt{2} g^2}{8M_W^2} = 1.1663788(6) \times 10^{-5} \text{GeV}^{-2}$ [PDG 2022] ⇒ Postdict M_W !

Example Weak Leptonic Decay $l^- \rightarrow e^- \nu_l \bar{\nu}_e$: $\Gamma_l \propto \left| \begin{array}{c} \nu_l \\ \bar{\nu}_e \\ e \\ l \end{array} \right|_{G_F}^2 = \frac{G_F^2 m_l^5}{192\pi^3}$ **Sargent's Rule** (dim. an.!).

Prediction $\frac{\Gamma_\tau m_\mu^5}{\Gamma_\mu m_\tau^5} \stackrel{!}{=} 1 \iff \text{exp: } 0.999 \pm 0.003$ confirms **Lepton Universality**

Corrections by Taylor & Quantum Effects

suppressed in powers of $\frac{\text{typ. low-momentum}}{\text{breakdown scale}} = \sqrt{\frac{q^2}{M_W^2}} \ll 1$

⇒ limited range of applicability $q^2 \ll M_W^2 \iff \text{range} \gg \frac{1}{M_W} \approx 0.002 \text{fm}$ very short-distance!

Fermi's V-A Effective (Low-Energy) Field Theory of GSW

W^\pm couples only to left-handed fermions/right-handed anti-fermions

\implies weak microscopic current $J_{\text{weak}}^\mu = \bar{u}_L \gamma^\mu u_L \propto \bar{u} \gamma^\mu (1 - \gamma_5) u$: (polar) **V**ector **M**inus **A**xial (vector)

Fermi's V-minus-A Theory (Model)

Fermi 1935 predates GSW by 35 years

Nature Rejection 1933: "contained speculations too remote from reality to be of interest to the reader".

Consequence: Fermi re-evaluates theory career, tries exp. Chicago Reactor. Nobel 1938.

Confinement & Hadronisation shield details of quark $\rightarrow W^\pm$ decays inside hadrons. \implies V-A modified.

$J_{\text{weak}}^\mu = g_V \bar{u}_L \gamma^\mu u_L - g_A \bar{u}_L \gamma^\mu \gamma_5 u_L$: couplings g_V, g_A depend on hadron.

\implies **Conserved Vector Current CVC**

$\bar{u}_L \gamma^\mu u_L$ baryon number must be conserved

Partially Conserved Axial Current PCAC

$\bar{u}_L \gamma^\mu \gamma_5 u_L$ not (fully) conserved

PCAC hypothesis (includes Sakurai) predates GSW

Prediction for neutron decay calculation not trivial: $\frac{g_A}{g_V} = \frac{5}{3} + \text{corrections} \iff 1.2754(13)$ [PDG 2022]

Weak interaction can serve as indirect probe of Physics at $\gtrsim 0.002\text{fm}$:

"Inside-Out Microscope" of QCD & Beyond-Standard-Model

(g) Universality for Quarks

Quark Hypercharges: from $Y_q = 2[Q_q - T_{3q}] \implies$ doublets $\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ have $Y_{qL} = +\frac{1}{3}$

$u_R, d_R, c_R, s_R, t_R, b_R$ have $Y_{qR} = 2Q_q \neq 0 \implies$ quark-equivalents of neutrinos do couple to $U(1)_Y!$

Anti-quarks of opposite helicity have opposite hypercharge.

Quark Decays: $\pi^+(u\bar{d}) \rightarrow \pi^0 e^+ \bar{\nu}_e \iff K^+(u\bar{s}) \rightarrow \pi^0 e^+ \bar{\nu}_e$

If universal, should differ only by "phase space". \implies Should extract $G_{\text{hadr}}^2 \approx 192\pi^3 \frac{\Gamma}{m_{\text{hadr}}^5} \stackrel{!}{=} G_F^2$!?!

exp: no at all! $G_{\pi^+} \approx 10 G_{K^+}$ and $G_{\pi^+}, G_{K^+} \neq G_F = 1.16 \dots \times 10^{-5} \text{ GeV}^{-2}$

Postulate common coupling to save quark universality: $G_F \stackrel{!}{=} \frac{G_{\pi^+}}{\cos \theta_C} \stackrel{!}{=} \frac{G_{K^+}}{\sin \theta_C}$: **Cabbibo angle** θ_C

exp: The 2 constraints hold. $\sin \theta_C = 0.2265(5)$: $\theta_C = 13.091(27)^\circ$ mnemonic: $\sin^2 \theta_W \approx \sin \theta_C$

\implies Leptonic & semi-leptonic weak couplings are related, and we restored & enlarged universality! \checkmark

Re-definition looks like a rotation of quark/hadron couplings: Change of basis?!?!?!:

\implies **Postulate:** Eigenstates q^{weak} of electro-weak T_3 (coupling to $\gamma W^\pm Z^0$)
are **not** eigenstates q^{mass} to mass operator \hat{M} (coupling to Higgs):

$[T_3, \hat{M}] \neq 0$ Not forbidden \implies Compulsory!

Two-Generation Quark Mixing: $(ud), (cs)$ and the GIM Mechanism

⇒ **Postulate:** Eigenstates q^{weak} of electro-weak T_3 (coupling to $\gamma W^\pm Z^0$)
 are **not** eigenstates q^{mass} to mass operator \hat{M} (coupling to Higgs):

One Can Show: Mathematically, one can choose $u_L^{\text{weak}} = u_L^{\text{mass}}$ etc. for upper components of doublet.

Mathematically sufficient to have the *lower* components of the weak doublets mix in flavour space:

$$\begin{pmatrix} d_L^{\text{weak}} \\ s_L^{\text{weak}} \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d_L^{\text{mass}} \\ s_L^{\text{mass}} \end{pmatrix}$$

⇒ Weak eigenstates of doublet couple to γ, W^\pm, Z^0 as before, but mass eigenstates mix, e.g.:

Weak Eigenstates $W_\mu \bar{u}_L \gamma^\mu d_L^{\text{weak}} = W_\mu [\bar{u}_L \gamma^\mu \cos \theta_C d_L^{\text{mass}} + \bar{u}_L \gamma^\mu \sin \theta_C s_L^{\text{mass}}]$
 couple via weak int.s: $W_\mu \bar{c}_L \gamma^\mu d_L^{\text{weak}} = W_\mu [\bar{c}_L \gamma^\mu (-\sin \theta_C) d_L^{\text{mass}} + \bar{c}_L \gamma^\mu \cos \theta_C s_L^{\text{mass}}]$

Consequence of Dictate of Universality: Works only if one weak-isospin partner for *each* quark!

⇒ New 4th quark *must* complete 2nd-generation doublet $\begin{pmatrix} \otimes_L \\ s_L \end{pmatrix}$ [Glashow/Iliopoulos/Maiani (GIM) 1970]
 [followed by J/ψ discovery 1973]

Two-Generation Quark Mixing: (ud) , (cs) and the GIM Mechanism

⇒ **Postulate:** Eigenstates q^{weak} of electro-weak T_3 (coupling to $\gamma W^\pm Z^0$)
 are **not** eigenstates q^{mass} to mass operator \hat{M} (coupling to Higgs):

One Can Show: Mathematically, one can choose $u_L^{\text{weak}} = u_L^{\text{mass}}$ etc. for upper components of doublet.

Mathematically sufficient to have the *lower* components of the weak doublets mix in flavour space:

$$\begin{pmatrix} d_L^{\text{weak}} \\ s_L^{\text{weak}} \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d_L^{\text{mass}} \\ s_L^{\text{mass}} \end{pmatrix}$$

⇒ Weak eigenstates of doublet couple to γ, W^\pm, Z^0 as before, but mass eigenstates mix, e.g.:

Weak Eigenstates couple via weak int.s:

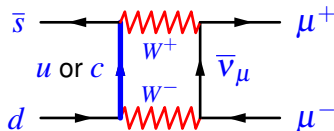
$$W_\mu \bar{u}_L \gamma^\mu d_L^{\text{weak}} = W_\mu [\bar{u}_L \gamma^\mu \cos \theta_C d_L^{\text{mass}} + \bar{u}_L \gamma^\mu \sin \theta_C s_L^{\text{mass}}]$$

$$W_\mu \bar{c}_L \gamma^\mu d_L^{\text{weak}} = W_\mu [\bar{c}_L \gamma^\mu (-\sin \theta_C) d_L^{\text{mass}} + \bar{c}_L \gamma^\mu \cos \theta_C s_L^{\text{mass}}]$$

Consequence of Dictate of Universality: Works only if one weak-isospin partner for *each* quark!

⇒ New 4th quark *must* complete 2nd-generation doublet $\begin{pmatrix} \otimes_L \\ s_L \end{pmatrix}$ [Glashow/Iliopoulos/Maiani (GIM) 1970]
 [followed by $J/\psi(c\bar{c})$ discovery 1973]

GIM Mechanism: flavour-changing neutral currents suppressed, e.g. $K^0(d\bar{s}) \rightarrow W^+W^- \rightarrow \mu^+\mu^-$:



$$d \xrightarrow{\cos \theta_C} u \xrightarrow{\sin \theta_C} s$$

$$d \xrightarrow{-\sin \theta_C} c \xrightarrow{\cos \theta_C} s$$

Equal in magnitude, opposite in sign.

⇒ Amplitudes cancel!

tiny nonzero because $m_u \neq m_c$

(h) Mixing for Three Generations: One Can Show

– Most general form allows upper entries of weak doublet to be eigenstates to *both* mass *and* weak:

– Most general matrix which mixes lower entries:

$$\underbrace{\begin{pmatrix} d_w \\ s_w \\ b_w \end{pmatrix}}_{\text{weak eigenstates}} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{Cabibbo-Kobayashi-Maskawa (CKM) matrix – includes Cabibbo matrix}} \underbrace{\begin{pmatrix} d_m \\ s_m \\ b_m \end{pmatrix}}_{\text{mass eigenstates}}$$

$u_w = u_m \quad c_w = c_m \quad t_w = t_m$

W/oLOG parametrised by 3 magnitudes + 1 complex phase: CP-violation in K^0/\bar{K}^0 ($\delta \approx 70^\circ$):

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97401(11) & 0.22650(48) & 0.00361^{(11)}_{(9)} \\ 0.22636(48) & 0.97320(11) & 0.04053^{(9)}_{(83)} \\ 0.00854^{(23)}_{(16)} & 0.03978^{(82)}_{(60)} & 0.999172^{(24)}_{(35)} \end{pmatrix} \quad \begin{array}{l} \text{From weak decays} \\ \text{of } N, K, B, D, \dots \\ \text{[PDG 2022]} \end{array}$$

– **Experiment:** Diagonal elements: coupling within same generation: ≈ 1

– **Experiment:** Off-diagonal elements much smaller: mixing generations 1 \longleftrightarrow 2: ≈ 0.2

Why that hierarchy?

mixing generations 2 \longleftrightarrow 3: $\approx 0.04 = 0.2^2$

mixing generations 1 \longleftrightarrow 3: $\approx 0.008 = 0.2^3$

Unitarity Test of the CKM matrix: Measure all matrix entries (including 3 complex phases).

– So far unitary \implies really 3 generations. If not: **New Quark Family/Beyond-Standard-Model??**

Assumed massless neutrinos. \implies No difference between mass and weak eigenstates.

But why should neutrinos be massless? – No compelling symmetry found.

Swiss Basic Law: Everything which is not forbidden, is compulsory.

Neutrino oscillations seen in solar, atmospheric, reactor & collider neutrino experiments: $m_\nu \sim eV$ ish

\implies Introduce analogue to CKM matrix, but now for *upper entries* of weak doublet (convenience).

$$\underbrace{\begin{pmatrix} \nu_{ew} \\ \nu_{\mu w} \\ \nu_{\tau w} \end{pmatrix}}_{\text{weak eigenstates}} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}_{\text{Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix: unitary, indep. parameters: 3 real \& 1 CP-violating phase } \delta = 245(32)^\circ} \underbrace{\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \\ \nu_{3m} \end{pmatrix}}_{\text{mass eigenstates}}$$

Much less diagonal that CKM (plus one complex phase, at present undetermined):

[PDG2022]

$$\begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} = \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.36 & 0.70 & 0.61 \\ 0.44 & 0.46 & 0.77 \end{pmatrix} \text{ — with errors } \pm [0.01 \dots 0.06]$$

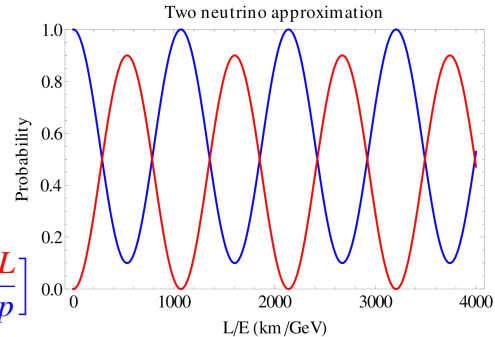
$$\begin{pmatrix} \nu_{ew} \\ \nu_{\mu w} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} \implies \text{time-evolution when only 1 species produced at } t = 0:$$

$$|\nu_{ew}(t)\rangle = \cos \theta e^{-iE_{\nu 1}t} |\nu_{1m}\rangle + \sin \theta e^{-iE_{\nu 2}t} |\nu_{2m}\rangle$$

Ultra-relativistic: $E_{\nu} = \sqrt{p^2 + m_{\nu}^2} \stackrel{E \gg m}{\approx} p \left(1 + \frac{m_{\nu}^2}{2p^2} \right)$

\implies Probability to find $|\nu_{ew}(t)\rangle$ at $L = \beta t = t$:

$$|\langle \nu_{ew}(L) | \nu_{ew}(0) \rangle|^2 = 1 - \sin^2 2\theta \sin^2 \left[\frac{\Delta m_{12}^2 = (m_1^2 - m_2^2) L}{4p} \right]$$



Disappearance Experiment: find remaining original \leftrightarrow **Appearance Experiment:** look for converted.

$\Delta m^2 \ll \text{eV}^2, p \gtrsim \text{MeV} \implies L \gg \frac{\text{MeV}}{\text{eV}^2} \sim \text{km}$: **QM interference on macroscopic lengths.**

Table 14.1: Sensitivity of different oscillation experiments.

Source	Type of ν	\bar{E} [MeV]	L [km]	$\min(\Delta m^2)$ [eV ²]
Reactor	$\bar{\nu}_e$	~ 1	1	$\sim 10^{-3}$
Reactor	$\bar{\nu}_e$	~ 1	100	$\sim 10^{-5}$
Accelerator	$\nu_{\mu}, \bar{\nu}_{\mu}$	$\sim 10^3$	1	~ 1
Accelerator	$\nu_{\mu}, \bar{\nu}_{\mu}$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric ν 's	$\nu_{\mu, e}, \bar{\nu}_{\mu, e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{-11}$

Besides θ , combination $\frac{\Delta m^2 L}{p = E}$ gives sensitivity:

Reactor: \oplus short L , controlled \ominus low- E

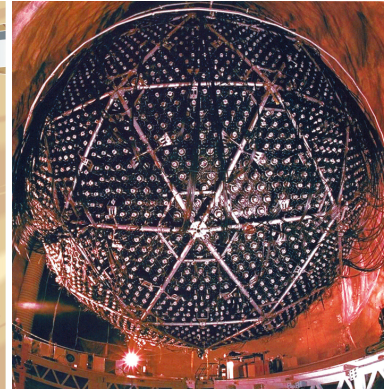
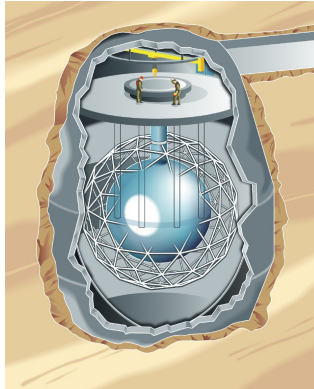
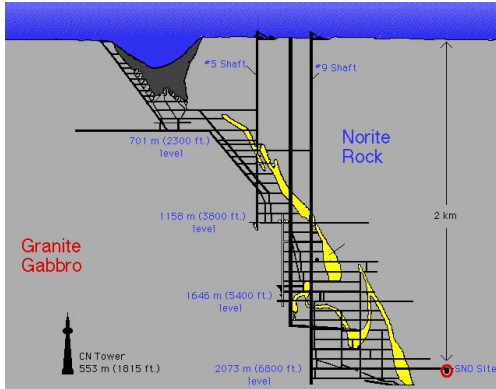
Accelerator: \oplus high- E , controlled \ominus short L

Atmospheric: \oplus high- $E, L = R_{\text{Earth}}$ \ominus no control

Solar: \oplus longest baseline \ominus solar modelling

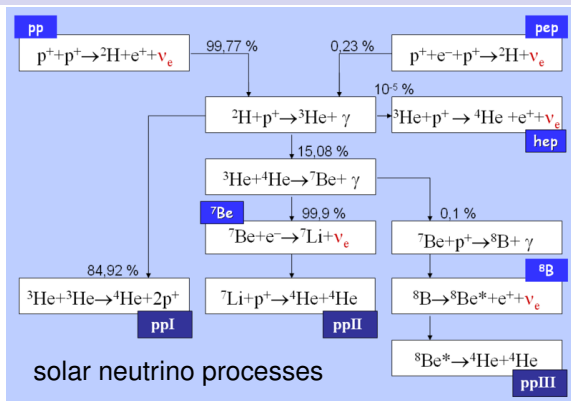
[PDG 2015]

Sudbury Neutrino Observatory SNO: Test Solar Neutrinos



1,000m³ D₂O, monitored by 9,600 Photomultipliers for Čerenkov light
2km under ground in operating nickel mine in Sudbury, Ontario, Canada.

SNO: Comprehensive Measurement of Neutrino Flux



Measure **total and individual solar neutrino flux** by Čerenkov of superluminal e^- of different origins:

Φ_e via $\nu_e d \rightarrow ppe^-$: breakup, omnidirectional: **CC**

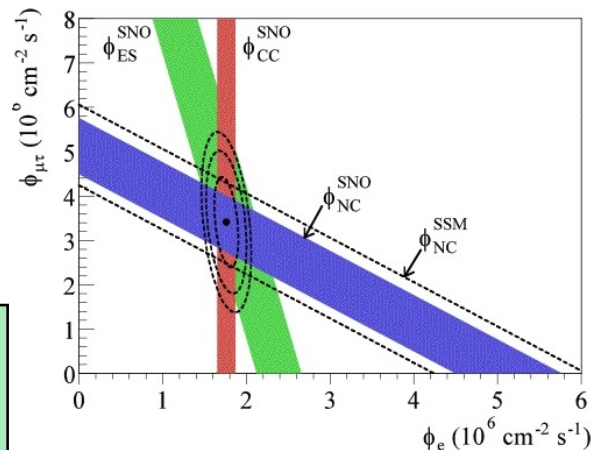
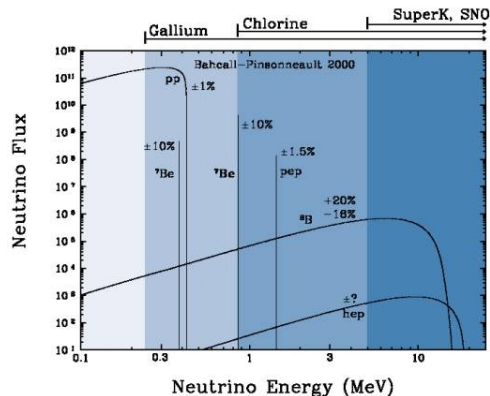
$\Phi_e + \Phi_{\mu\tau}$ via $\nu_e \mu\tau d \rightarrow pn\nu_e \mu\tau$: incl. scatt. **NC**
 $nd \rightarrow {}^3\text{H}\gamma(6\text{MeV}), \gamma e^- \Rightarrow e^-$ superlum.

$\Phi_e + 0.16\Phi_{\mu\tau}$ via $\nu_e \mu\tau e^- \rightarrow \nu_e \mu\tau e^-$: forward **ES**

Agrees excellently with Standard Solar Model!

$\Phi({}^8\text{B}) \propto \text{Temp}_{\text{Sun}}^{25} \leftrightarrow T_{\text{Sun}} = 15.7 \times 10^6 \text{K} \pm 1\%$.

$\theta_{12} = 33.6(8)^\circ, \Delta m_{12}^2 = 7.53(0.18) \times 10^{-5} \text{eV}^2$



Neutrino Oscillations: What We Know, What Not, and What's Cool

- Weak and mass eigenstates of neutrinos different. \implies Neutrinos mix.
- Neutrinos have nonzero mass-difference, ν_e is lightest.
- Is lightest neutrino massless? - What are the individual masses?
- Is $m_{\nu\mu} < m_{\nu\tau}$ (ordered like quark & charged-lepton masses), or $m_{\nu\mu} > m_{\nu\tau}$ ("inverted ordering")?

Majorana Neutrinos?

So far, $\nu = \begin{pmatrix} \text{particle}_R \\ \text{particle}_L \\ \text{antiparticle}_R \\ \text{antiparticle}_L \end{pmatrix}$

is **Dirac spinor**, but only ν_L and $\bar{\nu}_R$ couple.

Neutrinos charge-neutral, weak hypercharge is $Y = 0$.

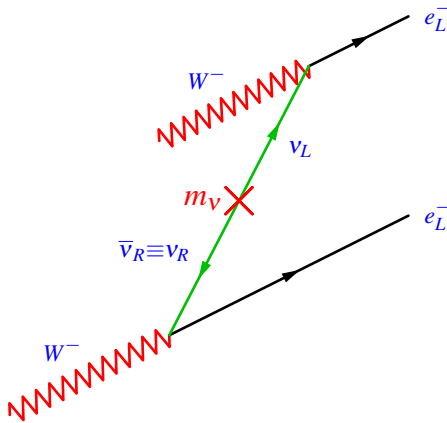
\implies **Could be its own antiparticle:** $\nu_R \equiv \bar{\nu}_R$, $\nu_L \equiv \bar{\nu}_L$

If so, then use that nonzero masses mix helicities e.g. in

$W^- \rightarrow e^- + (\bar{\nu}_R \equiv \nu_R)$: W^- decay

mass converts helicity: $\nu_R \xrightarrow{m_\nu \neq 0} \nu_R + \frac{m_\nu}{p_\nu} \nu_L$, $\frac{m_\nu}{p_\nu} \ll 1$

e^- production $\nu_L + W^- \rightarrow e^-$



\implies **Lepton Number violated by 2 units, probability $\propto \frac{m_\nu^2}{p_\nu^2}$!**

(i) Summarising Some Features of the GSW Theory

What We Like and Dislike About the GSW Theory

- 3 generations of quarks and leptons: nicely symmetric.

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad l_R \quad q_R$$

- Every particle but photon gets a mass: **Higgs-Kibble-Englert and Yukawa mechanisms**.
- **Unified** electromagnetic and weak interaction: 2 sides of same coin: first unification since Maxwell.
- **Universality** for *all* fermions.
- Has not failed any test yet – and we are really talking precision!
- But it took advantage of all freedoms ($W_\mu^0 B_\mu$ mixing, weak eigenstates, ν mass, ...)
- And why is parity violated in the first place?
- **Not nice**: not *one* coupling, but *two*: g, g' (or e, θ_W)
plus 2 Higgs parameters: VEV a & curvature λ ,
plus 2×4 CKM/PMNS mixing parameters,
plus 2×6 Higgs-fermion couplings to generate lepton & quark masses:

24 parameters is a lot!

QCD: 1 ($\alpha_s(M_Z^2) \longleftrightarrow \Lambda_{QCD}$) & 6 (double-counted) quark masses & 1 “vacuum angle” θ_{QCD}

- **Not nice**: Higgs the only “fundamental” scalar field in Nature – and why is its VEV nonzero??

(j) QCD vs. GSW

Both are **Quantum Field Theories**, and even **Gauge Theories**, and even **non-Abelian**.

⇒ Both show asymptotic freedom as $q^2 \rightarrow \infty$.

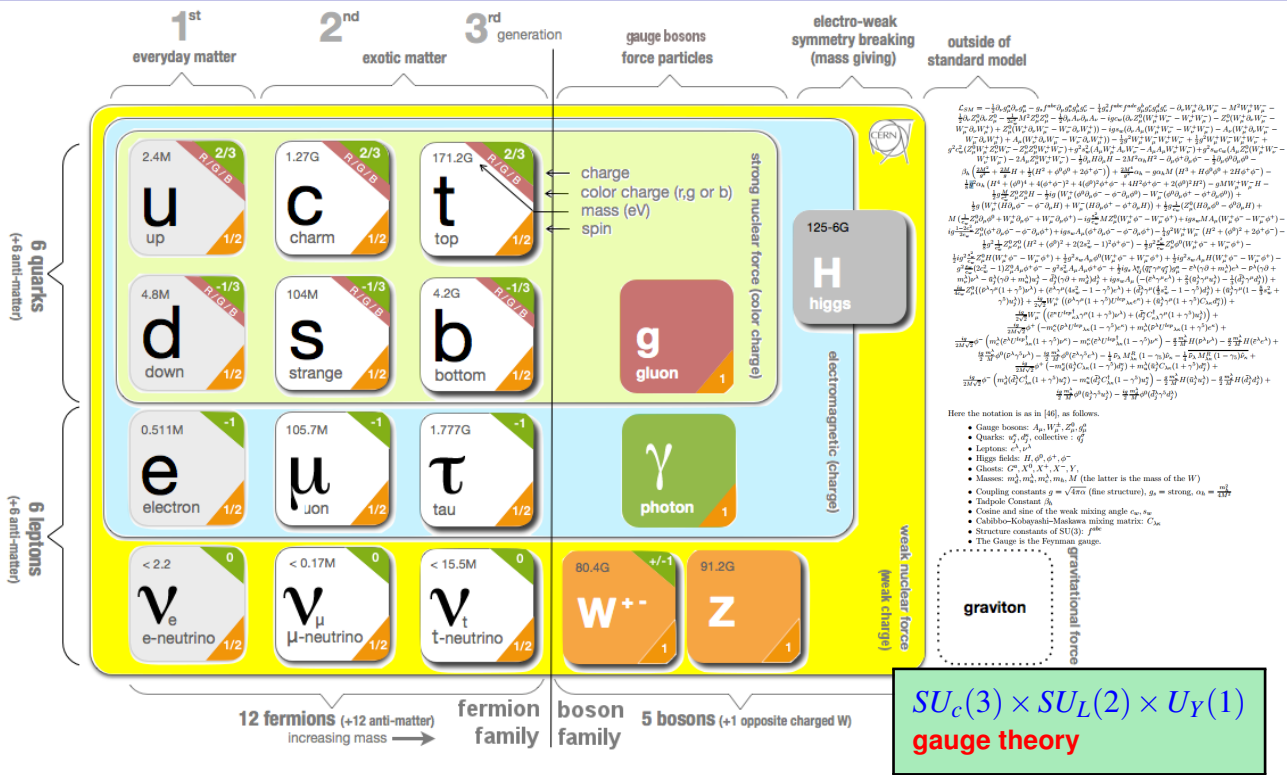
Obvious differences: only quark-gluon via colour $SU_c(3)$ vs. all particles via $SU_L(2) \times U_Y(1)$.

There are some oft-overlooked *fundamental* differences:

QCD (Confinement Phase)	GSW (Higgs Phase)
q & g confinement: not in detector	single leptons and gauge particles γ, W^\pm, Z^0 observed in detector
absence of coloured states	states with nonzero charge Q , hypercharge Y , weak isospin \vec{T} are common (e, τ, μ, ν, \dots)
nonperturbative at $q^2 \lesssim (3\text{GeV})^2$	perturbative everywhere
low-energy complicated: lattice, χ EFT	$q^2 \lesssim (30\text{GeV})^2$: EFT is simple Fermi/V-A
gluons massless (at least in perturbative régime)	3 of 4 gauge bosons massive by Higgs mechanism

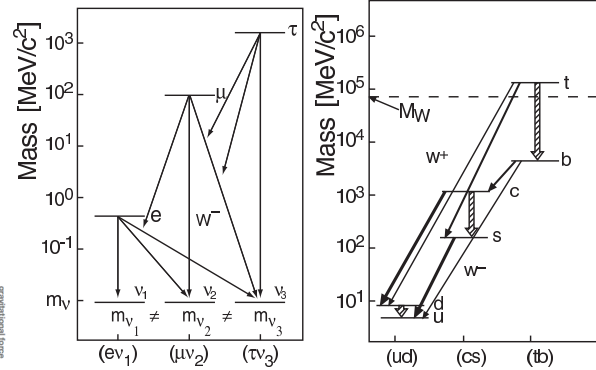
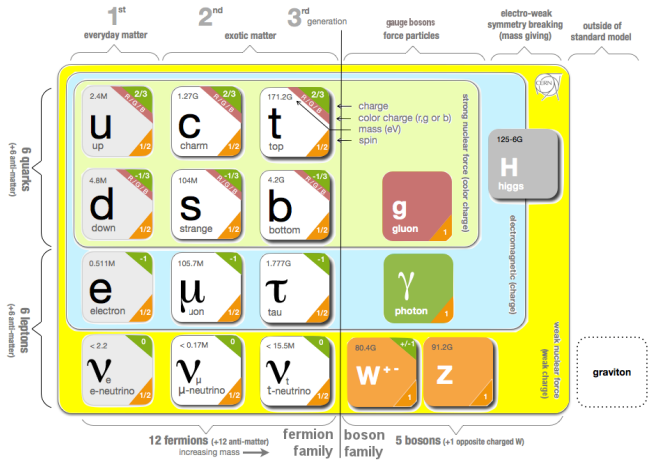
It's fair to say we do not understand why these are so different.

6. Finally: The Standard Model – and Beyond



This was a great time [. . .], the period of the famous triumph of quantum field theory. And what a triumph it was, in the old sense of the word: a glorious victory parade, full of wonderful things brought back from far places to make the spectator gasp with awe and laugh with joy. [S. Coleman 1985]

Answered a Lot of Questions, but Leave Many Open, For Example:



- Unification to 1 parameter
- Mass hierarchy problem
- Gravitation not quantised
- Why 3 generations? - Why $Q = \pm 1, \pm \frac{2}{3}, \pm \frac{1}{3}, 0$?
- Why these gauge groups? - Why 4 dimensions?

⇒ **Simplify (fewer parameters), or find processes which are not explained by freedoms of SM!**

Look for new fundamental particles (supersymmetry, strings, prions) & forces (dark energy/matter), violations of lepton & baryon number & universalities, Lorentz invariance, . . .

Lots of answers, but each raises more questions! ⇒ Your Turn!

But Wait, There is *More*: in PHYS 6710:

Nuclear and Particle Physics II: THE RETURN OF THE THEORIST

Topics Tuned To Audience; Typically: **Less-Formal QFT & Renormalisation**
– **Less-Formal Statistics & Data Analysis** – Instrumentation



Spring 2024 – watch this space!