PHYS 6610: Graduate Nuclear and Particle Physics I

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University

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II. Phenomena

4. Deep Inelastic Scattering and Partons

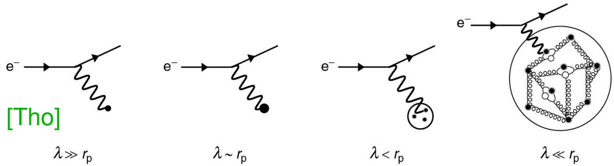
Or: Fundamental Constituents at Last

References: [HM 9; PRSZR 7.2, 8.1/4-5; HG 6.8-10]

(a) Inelastic Scattering $\to$ *Deep* Inelastic Scattering DIS

Breit/Brick-Wall Frame: no energy transfer $E - E' = 0$; momentum transfer maximal $\vec{p}'_{\text{Breit}} = -\vec{p}_{\text{Breit}}$.

Probe wave length $\lambda_{\text{Breit}} \sim \dfrac{1}{\sqrt{Q^2}}$ $\Longrightarrow$ **Dissipate energy and momentum into small volume** λ^3_{Breit}.



Now $Q^2 \gtrsim (3.5\text{GeV})^2 \sim (0.07\text{fm})^{-2} \gg r_N^{-2}$:

Energy cannot dissipate into whole N in $\Delta t \sim \dfrac{\lambda}{c}$

$\Longrightarrow$ Shoot hole into N, breakup dominates.

Deep Inelastic Scattering DIS $N(e^\pm, e')X$**: inclusive, i.e. all outgoing summed.**

Now characterised by 2 ***independent*** variables out of $(\theta, q^2 = -Q^2, E'_{\text{lab}}, \nu, W, x)$

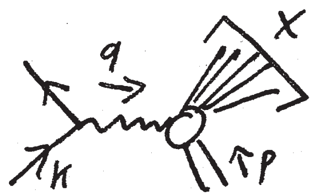
Lorentz-Invariant: $\nu = \dfrac{p \cdot q}{M} = E_{\text{lab}} - E'_{\text{lab}} > 0$ **energy transfer in lab**

Invariant mass-squared $W^2 = p'^2 = M^2 + 2p \cdot q + q^2 = M^2 + q^2(1-x)$

$\Uparrow$

Bjørken-$x = \dfrac{-q^2 = Q^2}{2p \cdot q = 2M\nu} \in [0;1]$: **inelasticity** (elastic scattering: $x = 1$)

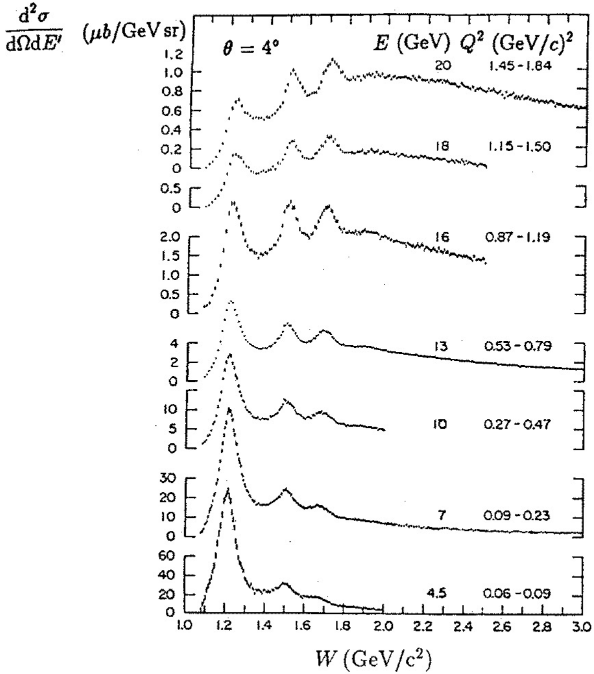
Dimension-less **structure functions** $F_{1,2}(x, Q^2)$ parametrise most general elmag. hadron ME.

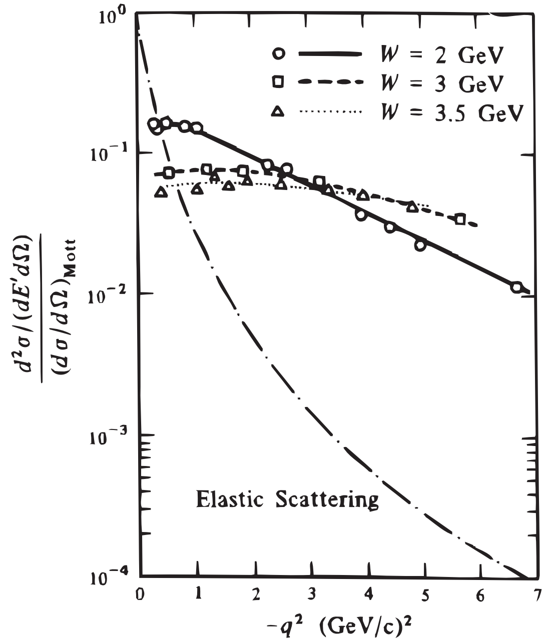


(b) Experimental Evidence

$E, Q^2 \nearrow$: **Resonances broaden & disappear into continuum for** $W \geq 2.5\,\mathrm{GeV}$

$\dfrac{\textbf{total}}{\textbf{Mott (elastic point)}}$ **depends only *weakly* on** Q^2 **at fixed** $W \gg M \implies$ **elastic on point constituents.**



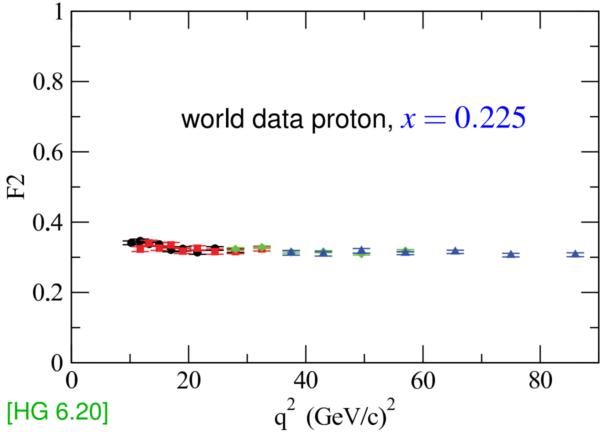


[HG 6.18]

Structure Functions $F_{1,2}$ are Q^2-Independent at Fixed Bjørken-x

$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\text{lab}} = \left(\frac{2\alpha}{Q^2}\right)^2 E'^2\cos^2\frac{\theta}{2}\left[\frac{F_2(Q^2,x)}{\nu}+\frac{2F_1(Q^2,x)}{M}\tan^2\frac{\theta}{2}\right] \quad \text{(I.7.6)}$$

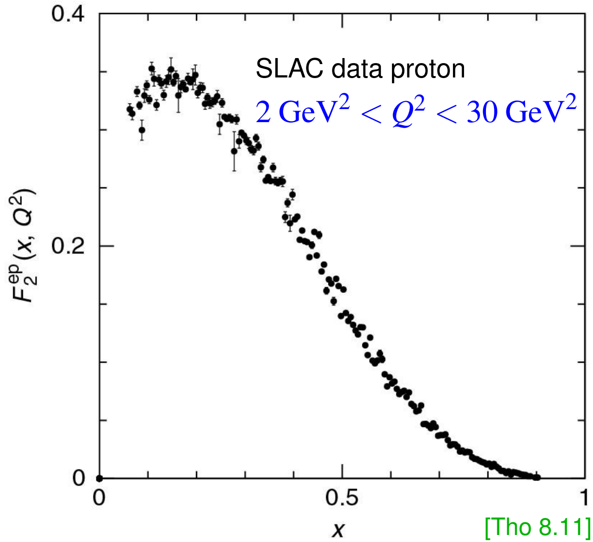
$F_{1,2}$ dimensionless, $Q^2, \nu \to \infty$ but $x = \dfrac{Q^2}{2M\nu}$ fixed: $F_{1,2}$ cannot dep. on Q^2, only on dimensionless x.



[HG 6.20]

Data: no new scale (e.g. mass, constituent radius)!

back-scattering events $\Longrightarrow F_1 \neq 0$: fermions.



[Tho 8.11]

$\Longrightarrow$ **Interpretation: Virtual photon absorbed by *charged, massless spin-$\frac{1}{2}$ point*-constituents:**

PARTONS. Idea: Bjørken 1967; name: Feynman 1969; soon identified with Gell-Mann's "quarks" of isospin.

(c) Sequence of Events in the Parton Model [HM 9, PRSZ]

Scaling: independence of Q^2 at fixed x.

***Not* a sign of QCD, but only that *no new scale* in nucleon: point-constituents.**

Scale-Breaking as sign of "small" interactions between constituents $\rightarrow$ QCD's DGLAP-WW (Part III)

$\Longrightarrow$ **DIS is *elastic* scattering on PARTONS: *charged,* $m \approx 0$ *spin-*$\frac{1}{2}$ *point*-constituents.**

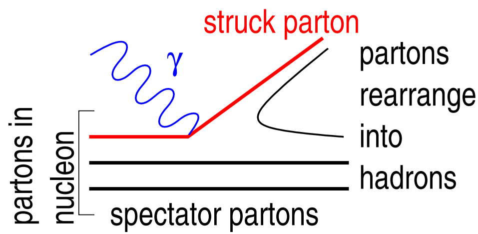
Problem: Partons not in detector $\longrightarrow$ Confinement hypothesis (later).

$\Longrightarrow$ **Assume that collision proceeds in two well-separated stages:**

(1) Parton Scattering: Timescale in Breit frame:

$$t_{\text{parton}} \approx \frac{\Delta x}{c} \approx \lambda \approx \frac{1}{Q} \ll 0.05 \frac{\text{fm}}{c} \text{ for } Q^2 \gg (4\text{GeV})^2.$$

$\Longrightarrow$ Photon interacts with *one* parton near-instantaneously, takes snapshot of parton configuration, frozen in time.



(2) Hadronisation: final-state interactions rearrange partons into hadron fragments, covert collision energy into new particles (inelastic!).

Much larger timescale $t_{\text{hadronisation}} \approx \dfrac{1}{\text{typ. hadron mass} \sim 1\text{GeV}} \approx 0.2\dfrac{\text{fm}}{c} \gg t_{\text{parton}}$.

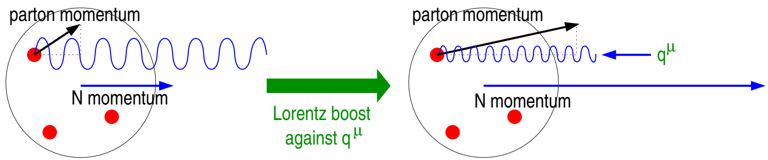
$\Longrightarrow$ Describe Scattering and Hadronisation independently of each other, no interference.

(d) Relating Elastic Parton Scattering & Nucleon DIS

What is the Bjørken-x? – The Infinite Momentum Frame

Problem: Transverse momenta $\vec{p}_q^{\,\perp}$ of partons sum to zero, but cannot simply infer them from p^μ!

Solution: Use $W, Q^2 \to \infty$ to boost along N-momentum axis $\vec{p}$ into **Infinite Momentum Frame IMF**.

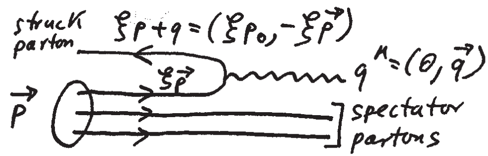


Transverse parton momenta unchanged, but longitudinal now $p_q^{\parallel} \xrightarrow{\text{boost}} \gamma p_q^{\parallel} \gg M, |\vec{p}_q^{\,\perp}|$.

$\Longrightarrow$ Transverse motion time-dilated: **Hadronisation** indeed much slower: rearranging by $\vec{p}_q^{\,\perp} \to 0$.

IMF is also a **Breit/Brick-wall frame**:

$\Longrightarrow$ Parton carries momentum fraction $0 \leq \xi \leq 1$ of total nucleon momentum.

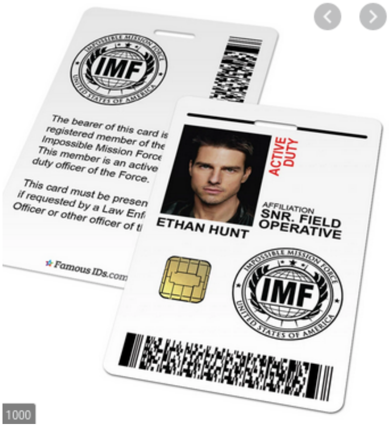


Assume Elastic Scattering on Parton: $(\xi p)^2 \stackrel{!}{=} (\xi p + q)^2 \Longrightarrow 2\xi p \cdot q + q^2 = 0$; $(\xi p)^2$ cancels.

$$\Longrightarrow \xi = \frac{-q^2}{2p \cdot q} = \frac{Q^2}{2M\nu} = x:$$

Bjørken-x $\widehat{=}$ fraction of hadron momentum which is carried by parton struck by photon in *Infinite Momentum Frame IMF*.

No, Not That IMF – And Not That IMF Either



International Monetary Fund (IMF)

Idea: DIS = *incoherent superposition* of elastic scattering in IMF on individual partons.

$\Longrightarrow$ Relate **Inelastic** to **Elastic Cross Section** $e\mu \to e\mu$ **on Point-Fermion** in lab frame:

elastic (I.7.4): $$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\mathrm{el}} = \left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}\left[1+\frac{Q^2}{2M^2}\,\tan^2\frac{\theta}{2}\right] \text{ with } E' = \frac{E}{1+\frac{E}{M}(1-\cos\theta)}$$

use $$-Q^2 \equiv q^2 = (k-k')^2 = -2k\cdot k' = -2EE'(1-\cos\theta) = -4EE'\sin^2\frac{\theta}{2} \qquad m_e \to 0!$$

$$\Longrightarrow \qquad \left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\mathrm{el}} = \left(\frac{2Z\alpha\,E'}{Q^2}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}\left[1+\frac{Q^2}{2M^2}\,\tan^2\frac{\theta}{2}\right]\delta\!\left[E' - \frac{E}{1+\frac{E}{M}(1-\cos\theta)}\right]$$

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use $$\frac{E'}{E}\,\delta[E'-\frac{E}{1+\frac{E}{M}(1-\cos\theta)}] = \underbrace{\frac{E'}{E}\left(1+\frac{E}{M}(1-\cos\theta)\right)}_{=1\text{ by }\delta\text{-distribution}}\delta[\underbrace{E'-E}_{=-\nu}+\underbrace{\frac{E\,E'}{M}(1-\cos\theta)}_{=Q^2/(2M)}]$$

$$=\delta[\nu-\frac{Q^2}{2M}] = \frac{1}{\nu}\,\delta[1-\frac{Q^2}{2M\nu}] = \frac{1}{\nu}\,\delta[1-\frac{Q^2}{2p\cdot q}] = \frac{1}{\nu}\,\delta[1-x]$$

as expected for elastic scattering ✓

$$\Longrightarrow \left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\mathrm{el}} = \left(\frac{2\alpha}{Q^2}\right)^2\cos^2\frac{\theta}{2}\;E'^2\,Z^2\left[\frac{1}{\nu}+\frac{Q^2}{2M^2\nu}\tan^2\frac{\theta}{2}\right]\delta[1-\frac{Q^2}{2p\cdot q}]$$

inelastic (I.7.6): $$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\mathrm{inel}} = \left(\frac{2\alpha}{Q^2}\right)^2\cos^2\frac{\theta}{2}\,E'^2\left[\frac{F_2(Q^2,x)}{\nu}+\frac{2\,F_1(Q^2,x)}{M}\tan^2\frac{\theta}{2}\right]$$

$\Longrightarrow$ F_2 of elastic on 1 point-fermion with momentum p only dep. on x: $F_2(Q^2,x) = Z^2\,\delta[1-\overbrace{\frac{Q^2}{2p\cdot q}}^{=x}]$

Idea: DIS $=$ *incoherent superposition* of elastic scattering in IMF on individual partons, each with charge Z_q and weighted by its *Parton Distribution Function PDF* $q(\xi)$:

probability density that scattered parton carries momentum fraction $[\xi;\xi+\mathrm{d}\xi]$ (in IMF).

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Compare Elastic and Inelastic Cross Sections more careful: [HM ex. 9.3ff; CL]

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$$\Longrightarrow\left.\frac{\mathrm{d}^2\sigma(Q^2,x=\frac{Q^2}{2p\cdot q})}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\mathrm{inel}}=\int\limits_0^1\mathrm{d}\xi\sum_{\text{all partons }q}q(\xi)\left.\frac{\mathrm{d}^2\sigma(\xi p)}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\substack{\text{elastic}\\\text{on parton}}}$$

Sum cross sections, no QM interference.

with $\delta[1-\frac{Q^2}{2p\cdot q}]\overset{p\to\xi p}{\longrightarrow}\delta[1-\frac{Q^2}{2\xi p\cdot q}]=\xi\;\underbrace{\delta[\xi-\frac{Q^2}{2p\cdot q}]}_{}$ and $M^2\equiv p^2\overset{p\to\xi p}{\longrightarrow}(\xi p)^2=\xi^2M^2$

$=\delta(\xi-x)$: parton momentum must match exp. kinematics

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$=\delta(\xi-x)$: parton momentum must match exp. kinematics

$$\left.\frac{\mathrm{d}^2\sigma(Q^2,x)}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\text{inel}}=\left(\frac{2\alpha}{Q^2}\right)^2\cos^2\frac{\theta}{2}\;E'^2\int\limits_0^1\mathrm{d}\xi\sum_{\text{all partons }q}Z_q^2\,q(\xi)\left[\frac{\xi}{\nu}+\frac{\xi}{\xi^2}\;\underbrace{\frac{Q^2}{2M^2\nu}}_{=x/M}\;\tan^2\frac{\theta}{2}\right]\delta(\xi-x)$$

| slaughter δ distribution

$$=\left(\frac{2\alpha}{Q^2}\right)^2\cos^2\frac{\theta}{2}\;E'^2\;\times\sum_{\text{all partons }q}Z_q^2\,q(x)\left[\frac{x}{\nu}+\frac{1}{M}\;\tan^2\frac{\theta}{2}\right]$$

Structure Functions F_2 and F_1 and Callan-Gross Relation

Idea: DIS $=$ *incoherent superposition* of elastic scattering in IMF on individual partons, each with charge Z_q and weighted by its *Parton Distribution Function PDF* $q(\xi)$: probability that scattered parton carries momentum fraction $[\xi;\xi+\mathrm{d}\xi]$ (in IMF).

$$\Longrightarrow \left.\frac{\mathrm{d}^2\sigma(Q^2,x)}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\text{inel}} = \left(\frac{2\alpha}{Q^2}\right)^2 \cos^2\frac{\theta}{2}\,E'^2 \sum_{\text{all partons } q} Z_q^2\,q(x)\left[\frac{x}{\nu}+\frac{1}{M}\,\tan^2\frac{\theta}{2}\right]$$

compare to $$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\text{inel}} = \left(\frac{2\alpha}{Q^2}\right)^2 \cos^2\frac{\theta}{2}\,E'^2 \left[\frac{F_2(Q^2,x)}{\nu}+\frac{2\,F_1(Q^2,x)}{M}\,\tan^2\frac{\theta}{2}\right]$$

$$\Longrightarrow F_2(Q^2,x) = \sum_{\text{all partons } q} Z_q^2\,x\,q(x) \quad \textbf{and} \quad F_1(Q^2,x) = \sum_{\text{all partons } q} \tfrac{1}{2}\,Z_q^2\,q(x)$$

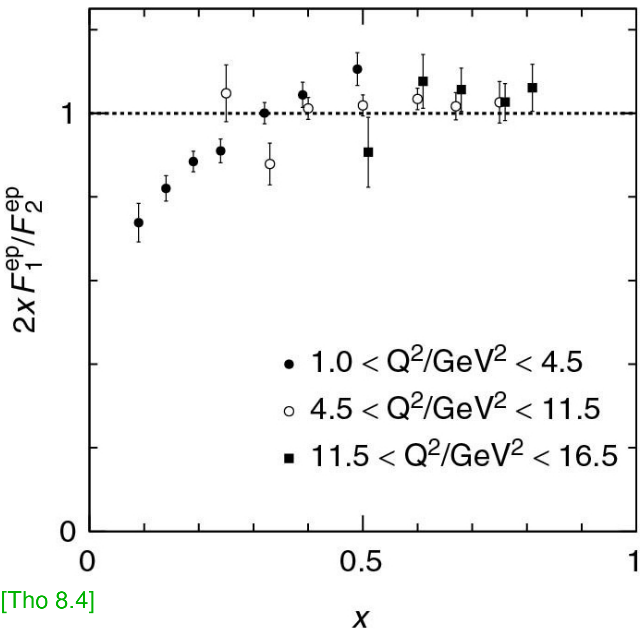
with $F_2(x) = 2x\,F_1(x)$ *Callan-Gross Relation*

All Q^2-independent and just result of incoherent scattering on point-fermions.

Each parton q has its own charge Z_q and Parton Distribution Function $q(x)$: $Z_u, u(x)$; $Z_d, d(x)$;...

Aside: Resist temptation to interpret $(\xi p)^2 = \xi^2 M^2$ as parton mass: depends on x, i.e. on kinematics!

Callan-Gross: Evidence for Point-Fermions in Nucleon



[Tho 8.4]

Expect for DIS ($Q^2, W^2 \to \infty,\ x$ fixed finite):

Callan-Gross Relation $2x\,F_1(x) = F_2(x)$

with $F_2(x) = x \sum_q Z_q^2\, q(x)$

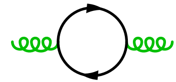
just from scattering on point-fermions.

Experimentally verified; corrections from QCD.

(e) Constituents of the Nucleon in the Parton Model

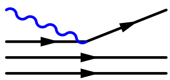
Quarks probability distrib. $u(x), d(x), s(x), \dots$ of quark flavour with momentum fraction x (in IMF).

Antiquarks with PDFs $\bar{u}(x), \dots$ only via vacuum fluctuations: virtual $q\bar{q}$ pairs.



Neutral Partons gluon PDF $g(x)$ carries momentum, spin, angular momentum,...

$\Longrightarrow$ **Valence Quarks** $q_v(x) := q(x) - \bar{q}(x)$ cannot disappear. $\Longrightarrow$ Follow initial quarks to detector.



Valence quarks carry *some (not all)* nucleon properties: baryon number, charge.
(still: these are *not the constituent quarks!*)

norm: $\displaystyle\int_0^1 \mathrm{d}x\ [u^N(x) - \bar{u}^N(x)] = \int_0^1 \mathrm{d}x\ u_v^N(x) = \begin{cases} 2 \text{ in proton (uud)} \\ 1 \text{ in neutron (ddu)} \end{cases}$; $\displaystyle\int_0^1 \mathrm{d}x\ d_v^N(x) = \begin{cases} 1 \text{ in p (uud)} \\ 2 \text{ in n (ddu)} \end{cases}$

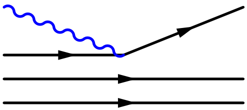
$\Longrightarrow$ **Sea Quarks** $\bar{q}_s(x) = \bar{q}(x)$, $q_s(x) = q(x) - q_v(x)$: All that is not valence.

sea created in $q\bar{q}$ pairs $\Longrightarrow q_s(x) \neq \bar{q}_s(x)$, but norm: $\displaystyle\int_0^1 \mathrm{d}x\ [q_s(x) - \bar{q}_s(x)] = 0 = \int_0^1 \mathrm{d}x\ \mathrm{sea}(x)$

$$\Longrightarrow \frac{1}{x} F_2^N(x) = \sum_q Z_q^2\, q^N(x) = \frac{4}{9}\left[u^N(x) + \bar{u}^N(x)\right] + \frac{1}{9}\left[d^N(x) + \bar{d}^N(x) + s^N(x) + \bar{s}^N(x)\right] + \dots + 0\, g(x)$$

$$= \underbrace{\frac{4}{9}\, u_v^N(x) + \frac{1}{9}\, d_v^N(x)}_{\text{valence contribution}} + \underbrace{\frac{4}{9}\left[u_s^N(x) + \bar{u}_s^N(x)\right] + \frac{1}{9}\left[d_s^N(x) + \bar{d}_s^N(x) + s_s^N(x) + \bar{s}_s^N(x)\right] + \dots}_{\text{sea contribution sea}(x)}$$

What PDFs to Expect – QUALITATIVELY!



only constituents:

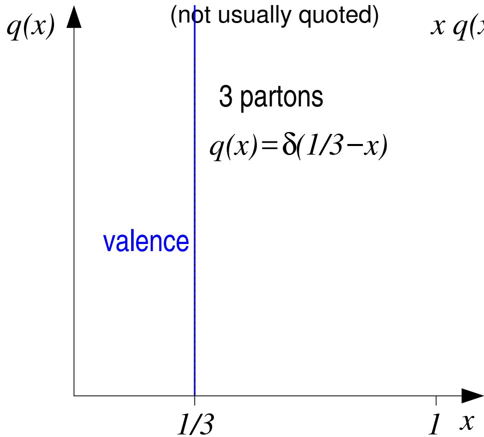
3 non-interacting

valence quarks

(constituent-q picture)

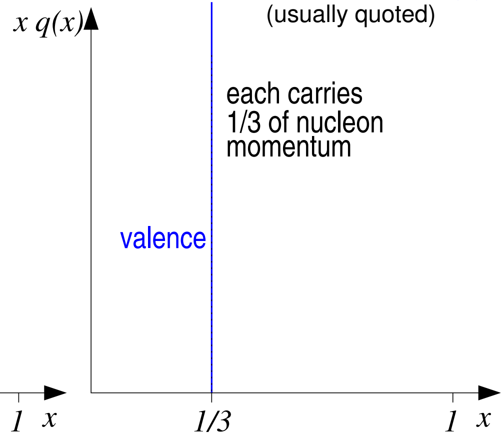
quark-distribution

(not usually quoted)



momentum-distribution $x\,q(x)$

(usually quoted)



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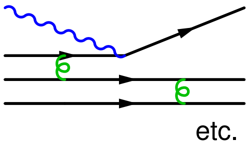
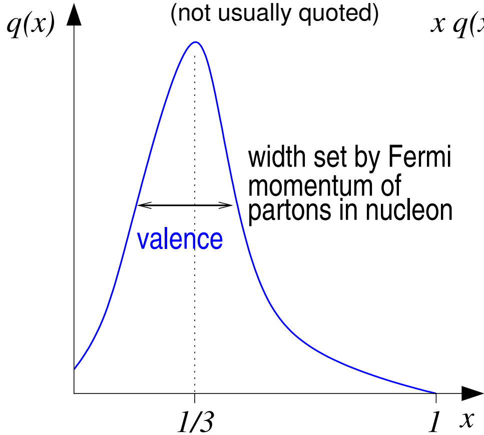
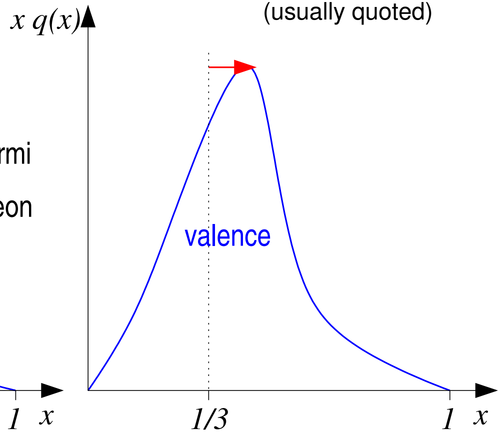


Add **instantaneous interactions**: distribute momentum & energy

But all momentum still carried by valence quarks.





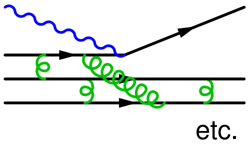
Fermi motion broadens peak

Maximum of momentum distribution $x_{\max}$ shifted to *right*:

$$\frac{\mathrm{d}}{\mathrm{d}x}[x_{\max}q(x_{\max})] = q(x_{\max}) + x_{\max}\underbrace{\frac{\mathrm{d}q(x_{\max})}{\mathrm{d}x}}_{=0} = q(x_{\max}) > 0$$

Momentum integral still $\sum\limits_q \int\limits_0^1 \mathrm{d}x\, x\, q(x) = 1$.

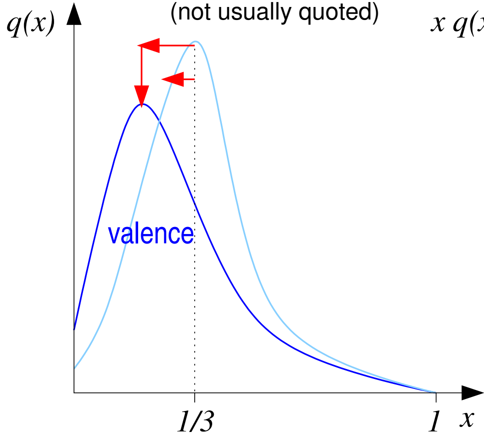
What PDFs to Expect – QUALITATIVELY!



Add **any** **interactions**: distribute momentum & energy to partons without charge (gluons)

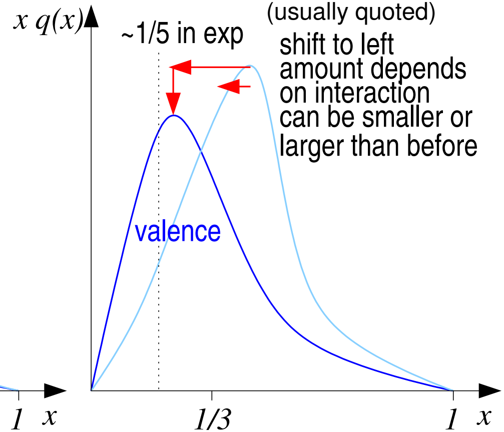
quark-distribution

(not usually quoted)



momentum-distribution $x\,q(x)$

(usually quoted)



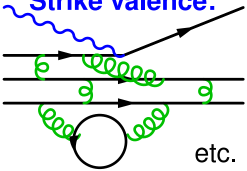
All maxima shifted to *left* – how much depends on *interactions*.

Momentum carried by valence quarks decreases.

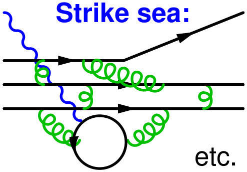
Still $\int q(x) = 1(2)$ **but momentum int. now smaller:** $\sum\limits_q \int\limits_0^1 \mathrm{d}x\, x\, q(x) < 1$.

What PDFs to Expect – QUALITATIVELY!

Strike valence:



Strike sea:



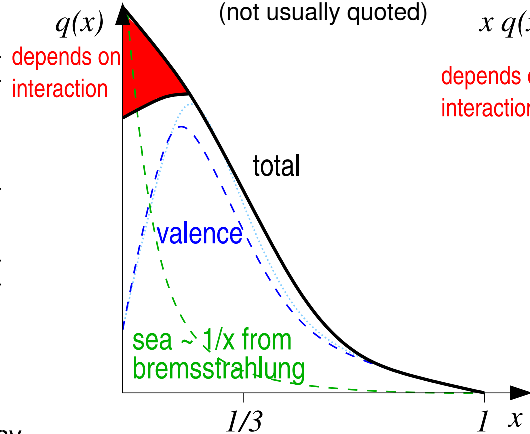
Add $q\bar{q}$ **sea**:

Take momentum away again from valence and couple to photon.

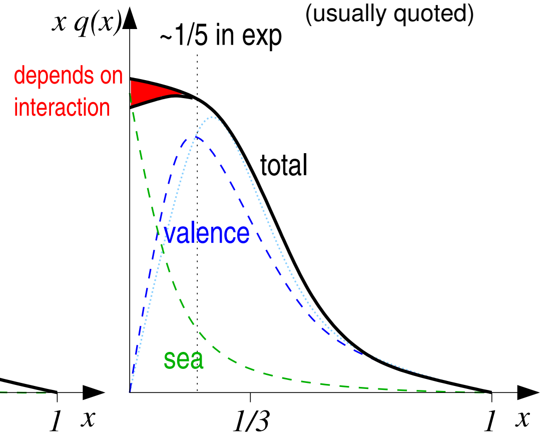
$\Longrightarrow$ most likely for small $x \mathrel{\hat{=}}$ small ξp

Momentum integral even smaller.

quark-distribution
(not usually quoted)



momentum-distribution $x\,q(x)$
(usually quoted)



Maxima again shifted to *right* – how much depends on *interactions*.

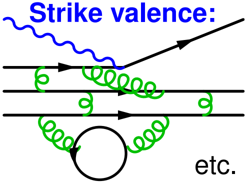
Expect **bremsstrahlung-like** spectrum $\sim \dfrac{1}{x}$ for **sea**, adds to **valence**.

$\Longrightarrow$ For $x \to 0$: $q(x)$ diverges, but mom. distribution $x\,q(x)$ **nonzero**.

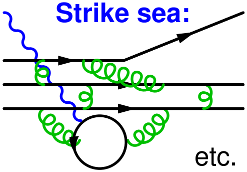
$\Longrightarrow$ **Small-x region particularly interesting to probe interactions with and between neutral constituents (glue).**

What PDFs to Expect – QUALITATIVELY!

Strike valence:



Strike sea:



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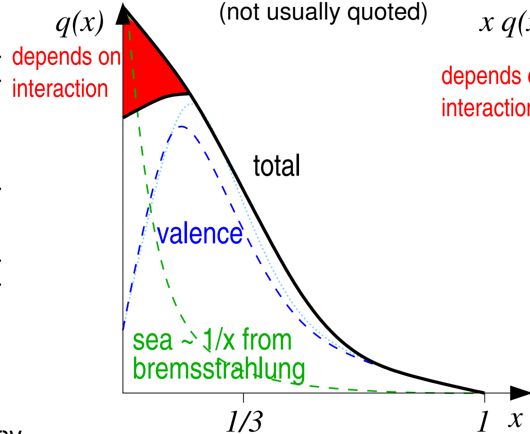
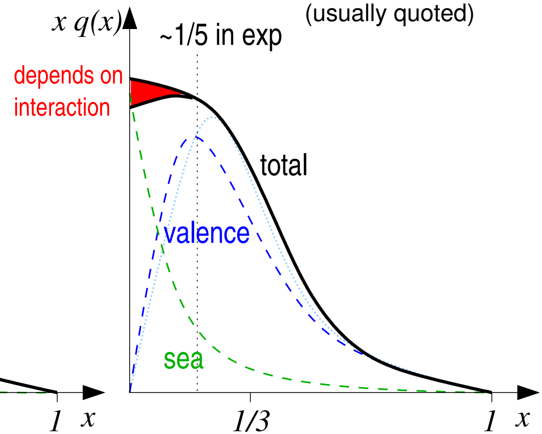
Momentum integral even smaller.

quark-distribution (not usually quoted)



momentum-distribution $x\,q(x)$ (usually quoted)



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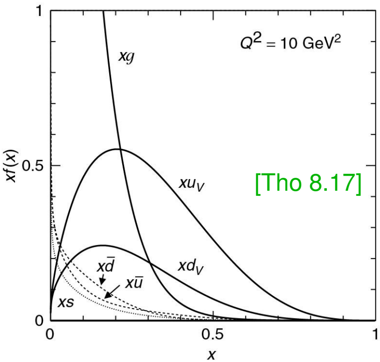
$\Longrightarrow$ **Small-x region particularly interesting to probe interactions with and between neutral constituents (glue).**

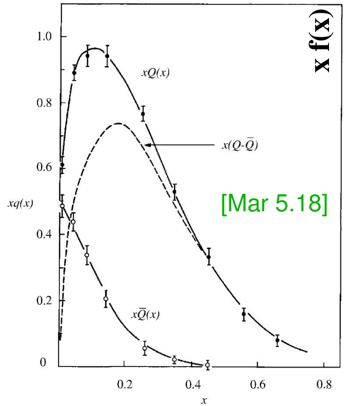
One cannot get the number of valence quarks from any peak position.

Books like [HM fig. 9.7, Per 5.8, Tho fig. 8.9] are WRONG!!

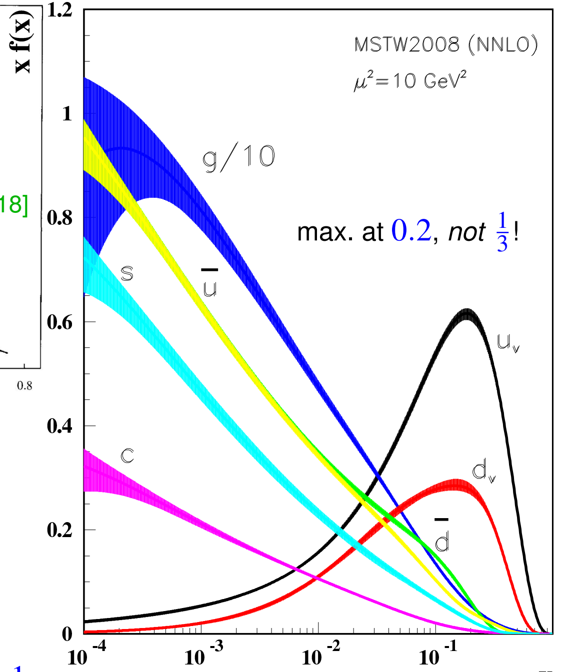
(f) What DIS Tells Us About Nucleon Structure: $x\,q(x)$



[Tho 8.17]



[Mar 5.18]



max. at 0.2, *not* $\frac{1}{3}$!

[PDG 2012 18.4: logarithmic scale!]

- **Valence quarks dominate as $x \gtrsim 0.5$**
- **Sea dominates for $x \to 0$: bremsstrahlung**
- **Peaks of F_2 and $q(x)$ *not* at $\frac{1}{3}$ but 0.17 & 0.2.**

$\Longrightarrow$ **Interactions in nucleon, neutral constituents.**

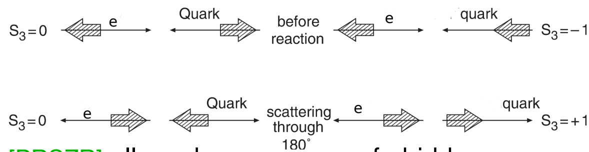
Sum Rules: e.g. momentum: $\int_0^1 \mathrm{d}x\, x[g(x) + \sum_q q(x)] = 1$

observable	valence	sea	gluons	other	total
nucleon momentum	31%	17%	52%	——	100%
nucleon spin	$[30\dots50]\%$	$\sim 0\%?$	lots%	orbital ang mom.	100% **(fake)** "spin crisis"

How To Dis-Entangle Parton Distributions

(1) **By target:** p vs. n (deuteron), ^{3}H etc. $\Longrightarrow$ PDFs inside proton vs. neutron etc.

(2) **By helicity:** polarised beam & ejectile: $\vec{e}N \to \vec{e}X$ via virt. γ **selects parton helicity/spin**

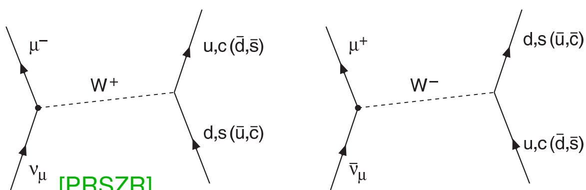


[PRSZR] allowed forbidden

(3) **By neutrinos:** $\nu N \to e^- X$, $\bar{\nu} N \to e^+ X$, $\nu N \to \nu X$, $\bar{\nu} N \to \nu X$: already 100% polarised helicity

weak int. $\Longrightarrow$ **different linear combinations of $q(x)$ and $\bar{q}(x)$**, **selects quark flavour & helicity**

Trick: use “invisible” neutrino beam, detect muons (no neutrals!)



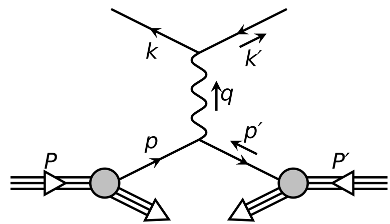
[PRSZR]

(4) **By “Drell-Yan process”:** use strong interactions in $NN \to X$

$\Longrightarrow$ can now see glue $g(x)$ directly, and $q(x), \bar{q}(x)$.

Gluon PDFs: integrals from sum rules,

e.g. momentum $\displaystyle\int \mathrm{d}x\, x\, g(x) = 1 - \sum_q \int \mathrm{d}x\, x\, q(x)$.



Isospin: Proton uud vs. Neutron $ddu \Longrightarrow u^p(x) \stackrel{?}{=} d^n(x)$, $d^p(x) \stackrel{?}{=} u^n(x)$

$$\Longrightarrow u(x) := u^p(x) = d^n(x)$$
$$d(x) := d^p(x) = u^n(x)$$

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{4d_v + u_v + \text{sea}^n}{d_v + 4u_v + \text{sea}^p}$$

Low x: sea dominates, isospin-symmetric $\Longrightarrow \frac{F_2^n}{F_2^p} \approx \frac{\text{sea}^n}{\text{sea}^p} \to 1$ ✓ exp.

High x: valence dominates; data: $\Longrightarrow \frac{F_2^n}{F_2^p} \approx \frac{4d_v + u_v}{4u_v + d_v} \to \frac{1}{4}$ in exp.

explained if u_v carries more momentum than $d_v \leftrightarrow p = (uud)$ (*not* Coulomb!)

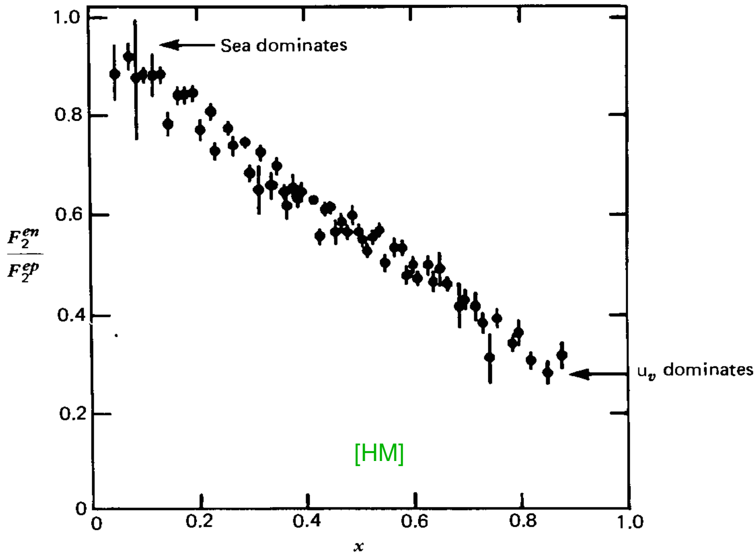


Fig. 9.6 The ratio F_2^{en}/F_2^{ep} as a function of x, measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator.

Another Sum Rule: $\frac{1}{x}[F_2^p(x) - F_2^n(x)] = \frac{1}{3}[u_v - d_v] + [\text{sea}^p - \text{sea}^n]$

Sea on average not quite isospin-symmetric (except at very low x).

Gottfried Sum Rule $\int\limits_0^1 \frac{\mathrm{d}x}{x}[F_2^p(x) - F_2^n(x)] = \frac{1}{3} + \int\limits_0^1 \mathrm{d}x\,[\text{sea}^p - \text{sea}^n]$

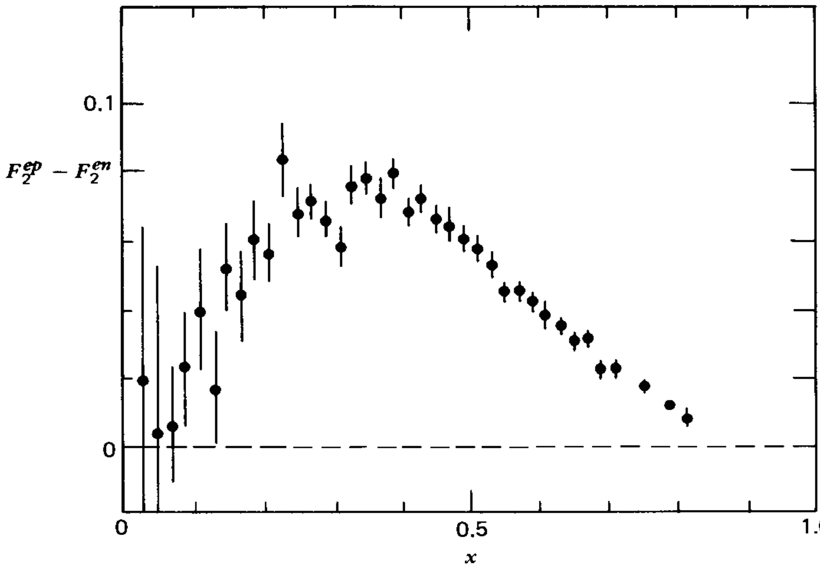


Fig. 9.8 The difference $F_2^{ep} - F_2^{en}$ as a function of x, as measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator. [HM]

Next Muon Collab. (CERN) at $Q^2 = (2\text{GeV})^2$:

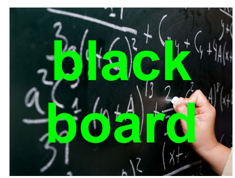
$G\Sigma R = [0.228 \pm 0.007]$

$\Longrightarrow \int\limits_0^1 \mathrm{d}x\,[\bar{d} - \bar{u}] \approx 0.16$

(assumes $s \approx \bar{s} \approx 0$)

$\Longrightarrow$ **Light sea on average not flavour-symmetric: $\bar{u} < \bar{d}$!**

$\rightarrow$ Meson Cloud Model...



(g) PDFs in Nuclei: The EMC Effect

EMC collaboration 1983
[PRSZR 8.5]

Regions where sea dominates:

$x \lesssim 0.06$: F_2^A significantly smaller than free

$0.06 \lesssim x \lesssim 0.3$: F_2^A slightly larger than free

Effects increase with A.

Regions where valence dominates:

$0.3 \lesssim x \lesssim 0.8$: F_2^A slightly smaller than free;
minimum at $x \approx 0.65$
$\Longrightarrow$ avg. momentum of bound partons smaller;
invest momentum in binding (gluons)?

$x \gtrsim 0.8$: $F_2^A \nearrow 1$: individual N

$x > 1$ possible: suck momentum from other N.

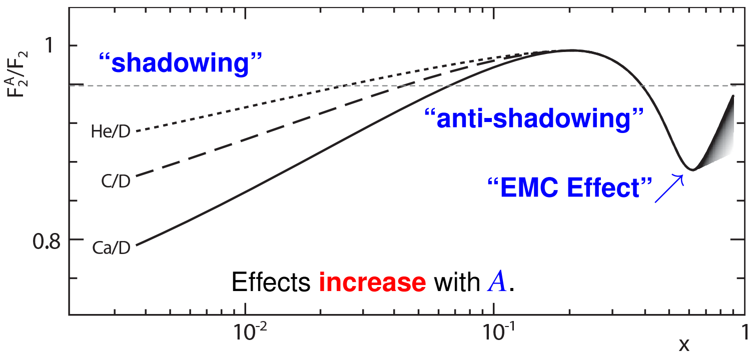


Fig. 7.8. Ratio of the structure functions F_2 of lithium, oxygen, and calcium over deuterium as a function of x [Ar88, Go94b, Am95]. [PRSZ, 6th ed.]

No Established Explanation Yet:

Multi-quark cluster?

qq-int. across nucleon boundaries?

$x \to 0$: resolution $\gtrsim \frac{1}{xp}$ bad $\Longrightarrow$
“Overcrowding”: nucleons in nucleus share sea?

“Nuclear Shadowing”: low-x photons react with surface by virtual mesons?

(h) Generalising Parton Distributions

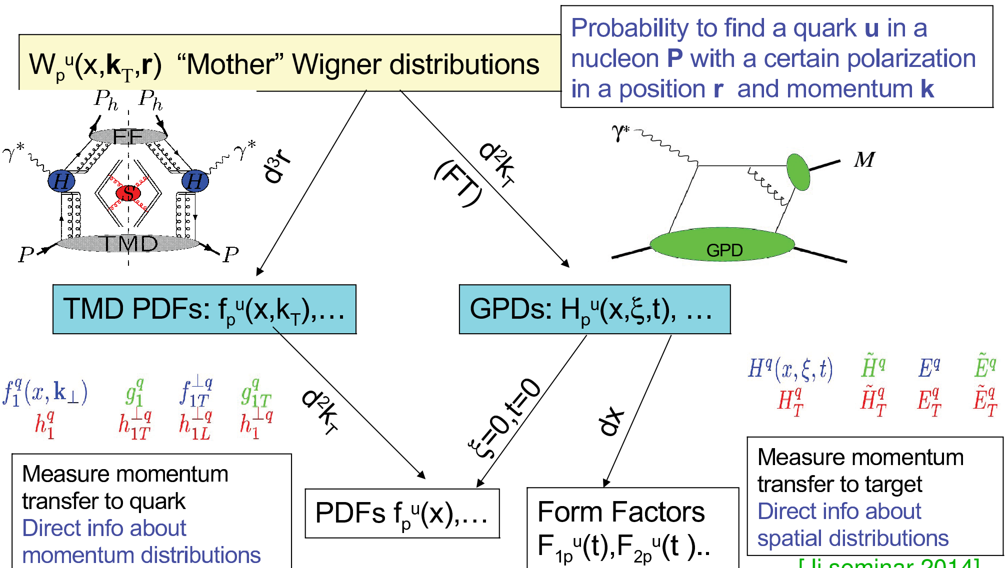
PDFs: $\vec{p}_{\text{parton}} = x\vec{p}$, transverse parton momentum negligible $\Longrightarrow q(x, Q^2)$

$\rightarrow$ **Extension:** add impact $b_q \leftrightarrow \vec{p}_{q\perp}$, q- & N-spin orientations, mom. transfer,. . .

$\Longrightarrow$ Most general parametrisation: over 20 functions, each with more than 5 parameters:

Wigner Distributions, including **Transverse Momentum Distributions TMDs** and **Generalised Parton Distributions GPDs**: fun for years to come. . .

Challenging; co-motivation for Jlab ugrade: many parameters, many functions, small effects.



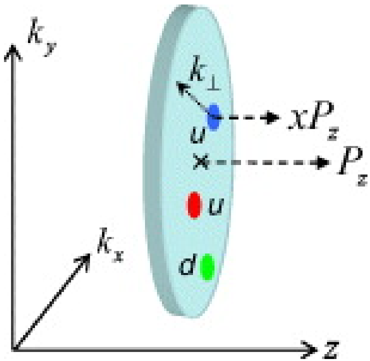
[Ji seminar 2014]

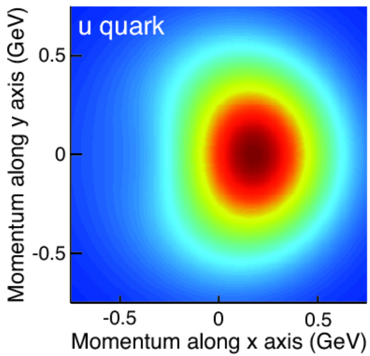
Example Transverse Momentum Distributions TMDs

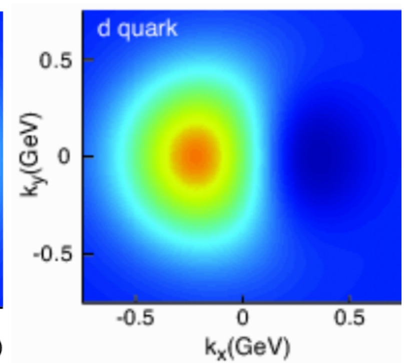
Add impact parameter $b_q \leftrightarrow \vec{p}_{q\perp}$ transverse quark momentum.

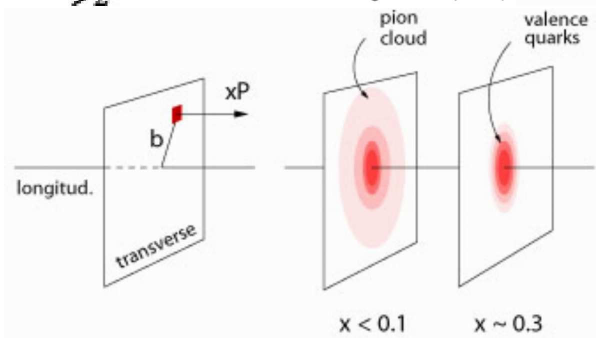
Interpretation: snapshot of quark distribution $q(x, b, Q^2)$ in nucleon **perpendicular** to $\vec{p}$. [Burkhardt 04]

"Nucleon Tomography"









Next: 5. Quarks in e^+e^- Annihilation

Familiarise yourself with: [PRSZR 9.1/3; PRSZR 15/16 (cursorily); HG 10.9, 15.1-7; HM 11.1-3; Tho 9.6]

Alt: Compare Elastic/Inelastic Cross Sections more careful: [HM ex. 9.3ff; CL]

Idea: DIS $=$ *incoherent superposition* of elastic scattering on individual partons in IMF.

$\Longrightarrow$ Relate **Inelastic** to **Elastic Cross Section** $e\mu \to e\mu$ **on Point-Fermion** in lab frame:

inelastic (I.7.6): $$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\mathrm{inel}} = \left(\frac{2\alpha}{q^2}\right)^2 \cos^2\frac{\theta}{2}\,E'^2\left[\frac{F_2(Q^2,x)}{\nu} + \frac{2\,F_1(Q^2,x)}{M}\tan^2\frac{\theta}{2}\right]$$

elastic (I.7.4): $$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\mathrm{el}} = \left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}\left[1-\frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$ with $E' = \dfrac{E}{1+\frac{E}{M}(1-\cos\theta)}$

use $$q^2 = (k-k')^2 = -2k\cdot k' = -2E\,E'(1-\cos\theta) = -4E\,E'\sin^2\frac{\theta}{2}$$

$$\Longrightarrow \quad \left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\mathrm{el}} = \left(\frac{2Z\alpha\,E'}{q^2}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}\left[1-\frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]\delta\!\left[E' - \frac{E}{1+\frac{E}{M}(1-\cos\theta)}\right]$$

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use $$\frac{E'}{E}\,\delta\Big[E'-\frac{E}{1+\frac{E}{M}(1-\cos\theta)}\Big] = \underbrace{\frac{E'}{E}\left(1+\frac{E}{M}(1-\cos\theta)\right)}_{=1\text{ by }\delta\text{-distribution}}\delta\Big[\underbrace{E'-E}_{=-\nu}+\underbrace{\frac{E\,E'}{M}(1-\cos\theta)}_{=-q^2/(2M)}\Big]$$

$$=\delta\Big[\nu+\frac{q^2}{2M}\Big] = \frac{1}{\nu}\,\delta\Big[1+\frac{q^2}{2M\nu}\Big] = \frac{1}{\nu}\,\delta\Big[1+\frac{q^2}{2p\cdot q}\Big] = \frac{1}{\nu}\,\delta[1-x]$$

expected for elastic scattering ✓

$$\Longrightarrow \left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\text{el}} = \left(\frac{2Z\alpha}{q^2}\right)^2 \cos^2\frac{\theta}{2}\,E'^2\left[\frac{1}{\nu}-\frac{q^2}{2M^2\nu}\tan^2\frac{\theta}{2}\right]\delta\Big[1+\frac{q^2}{2p\cdot q}\Big]$$

$\Longrightarrow$ F_2 of elastic on 1 point-fermion with momentum p only dep. on x: $F_2(Q^2,x) = Z^2\,\delta\Big[1+\overbrace{\frac{q^2}{2p\cdot q}}^{=-x}\Big]$

Alt: Compare Elastic/Inelastic Cross Sections more careful: [HM ex. 9.3ff; CL]

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Now incoherent, elastic scattering on individual partons with charges Z_q, each weighted by
Parton Distribution Function PDF **$q(\xi)$: probability parton has mom. fraction $[\xi;\xi+\mathrm{d}\xi]$ (in IMF).**

$$F_2(Q^2,x) = \underbrace{\int\limits_0^1 \mathrm{d}\xi}_{\text{all parton momenta}} \sum_{\text{all partons } q} Z_q^2\,\underbrace{q(\xi)}_{\text{PDF}}\,\delta[1+\frac{q^2}{2\xi\,p\cdot q}] = \int\limits_0^1 \mathrm{d}\xi \sum_{\text{all partons } q} Z_q^2\,\xi\,q(\xi)\,\underbrace{\delta[\xi+\frac{q^2}{2p\cdot q}]}_{=\delta(\xi-x)}$$

momentum must match exp. kinematics

$$\boxed{\Longrightarrow F_2(Q^2,x) = \sum_{\text{all partons } q} Z_q^2\,x\,q(x)}$$

Alt: Callan-Gross Relation Between Structure Functions F_2 and F_1

Idea: DIS $=$ *incoherent superposition* of elastic scattering on individual partons in IMF.

$$\Longrightarrow \left.\frac{\mathrm{d}^2\sigma(Q^2, x=\frac{-q^2}{2p\cdot q})}{\mathrm{d}\Omega\,\mathrm{d}E'}\right|_{\text{inel}} = \int_0^1 \mathrm{d}\xi \sum_{\text{all partons } q} q(\xi)\left.\frac{\mathrm{d}\sigma(\xi p)}{\mathrm{d}\Omega}\right|_{\substack{\text{elastic}\\ \text{on parton}}}$$

Sum cross sections, no QM interference.

Compare Hadronic Tensors: $$\boxed{|\overline{\mathcal{M}}|^2 \propto L_{\mu\nu}\,W^{\mu\nu} = L_{\mu\nu}\int_0^1 \mathrm{d}\xi \sum_{\text{all partons } q} w^{\mu\nu}_{\text{el. spin}\frac{1}{2}}(\xi p)\,q(\xi)}$$

inel.: $$W^{\mu\nu}_{\text{inel}}(q^2,x) = \frac{F_1(q^2,x)}{M}\left[\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right] + \frac{F_2(q^2,x)}{M^2\nu}\left[p^\mu - \frac{p\cdot q}{q^2}q^\mu\right]\left[p^\nu - \frac{p\cdot q}{q^2}q^\nu\right] \quad \text{(I.7.6W)}$$

elastic: $$w^{\mu\nu}_{\text{el. spin}\frac{1}{2}}(p) = 2Z^2\,[p^\mu(p+q)^\nu + (p+q)^\mu p^\nu - g^{\mu\nu}\,p\cdot q]\,\delta[1+\underbrace{\frac{q^2}{2p\cdot q}}_{=-x}] \quad \text{(I.7.4W)}$$

use $q^\mu L_{\mu\nu} = q^\nu L_{\mu\nu} = 0$ to drop terms $\propto q^\mu, q^\nu$

inelastic: $$W^{\mu\nu}_{\text{inel}}(q^2,x) \to \frac{F_2(q^2,x)}{M^2\nu}\,p^\mu\,p^\nu - \frac{F_1(q^2,x)}{M}\,g^{\mu\nu} + (q^\mu, q^\nu)\text{-terms}$$

elastic on **parton**: $$w^{\mu\nu}_{\text{el. spin}\frac{1}{2}}(\xi p) \to 2Z^2\,[2\,\xi p^\mu\,\xi p^\nu - g^{\mu\nu}\,\xi p\cdot q]\,\delta[1+\frac{q^2}{2\xi p\cdot q}] + (q^\mu, q^\nu)$$

Alt: Callan-Gross Relation Between Structure Functions F_2 and F_1

Idea: DIS $=$ *incoherent superposition* of elastic scattering on individual partons in IMF.

$$\Longrightarrow \frac{\mathrm{d}^2\sigma(Q^2, x=\frac{-q^2}{2p\cdot q})}{\mathrm{d}\Omega\,\mathrm{d}E'}\Bigg|_{\text{inel}} = \int\limits_0^1 \mathrm{d}\xi \sum_{\text{all partons } q} q(\xi)\, \frac{\mathrm{d}\sigma(\xi p)}{\mathrm{d}\Omega}\Bigg|_{\substack{\text{elastic}\\ \text{on parton}}}$$

Sum cross sections, no QM interference.

Compare Hadronic Tensors:

$$\boxed{|\overline{\mathcal{M}}|^2 \propto L_{\mu\nu}\, W^{\mu\nu} = L_{\mu\nu} \int\limits_0^1 \mathrm{d}\xi \sum_{\text{all partons } q} w^{\mu\nu}_{\text{el. spin}\frac{1}{2}}(\xi p)\, q(\xi)}$$

inelastic: $$W^{\mu\nu}_{\text{inel}}(q^2,x) \to \frac{F_2(q^2,x)}{M^2\nu}\, p^\mu\, p^\nu - \frac{F_1(q^2,x)}{M}\, g^{\mu\nu} + (q^\mu, q^\nu)\text{-terms} \qquad \text{(I.7.6W)}$$

elastic on **parton**: $$w^{\mu\nu}_{\text{el. spin}\frac{1}{2}}(\xi p) \to 2Z^2\,[2\,\xi p^\mu\, \xi p^\nu - g^{\mu\nu}\, \xi p\cdot q]\,\delta[1+\frac{q^2}{2\xi p\cdot q}] + (q^\mu, q^\nu)$$

Different mass-dimensions in $W^{\mu\nu}$ and $w^{\mu\nu}$ $\Longrightarrow$ cannot compare directly, but **ratios** must match:

$$\frac{p^\mu p^\nu\text{-term}}{g^{\mu\nu}\text{-term}} : \frac{2\xi}{p\cdot q} \overset{!}{=} \frac{F_2}{(M\nu = p\cdot q)}\,\frac{1}{F_1} \Longrightarrow$$

Callan-Gross Relation $F_2(x) = 2x\, F_1(x)$
Just result of scattering on point-fermions.

$$\Longrightarrow F_2(Q^2,x) = \sum_{\text{all partons } q} Z_q^2\, x\, q(x) \quad \text{and} \quad F_1(Q^2,x) = \sum_{\text{all partons } q} \frac{1}{2}\, Z_q^2\, q(x)$$