Problem Sheet 13

Due date: 26 April 2018 **12:00**

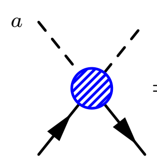
For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

1. Gell-Mann–Okubo vs. Gell-Mann–Oakes–Renner: The Quadratic-Mass Formula (**5P**): We saw that a linear relation between meson and quark mass does not work for the light meson octet. Before χEFT and QCD, people tried a quadratic fit, and it did surprisingly well. Now we may be able to understand this by applying the Gell-Mann–Okubo relation, $m^2_{\text{meson}} \propto m_{q1} + m_{q2}$ to constituent quarks and assuming that the constant of proportionality is system-independent (all that is dangerous, but it works...). This lends credence to the idea that the light meson octet would have zero mass in the chiral limit (Goldstone bosons).

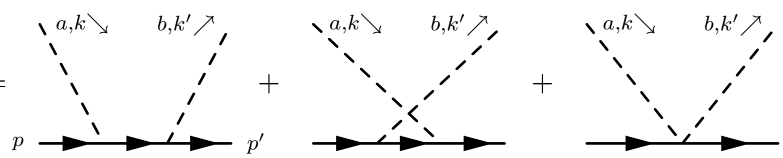
   Use isospin symmetry and the π^+ and K^+ meson masses to predict the ratio of the average u and d-quark masses to that of the s quark. Check against the PDG ratio.

2. The Pion-Nucleon Scattering Length in χEFT, Part I (**15P**): This is your "**Capstone Project**" for this course. I have outlined it in the lecture, but here are the details.

   At leading order in χEFT (Baron-χPT version), the amplitude has 3 parts, labelled for convenience.



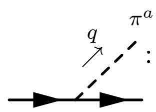
$= +\mathrm{i}\left[\delta^{ab}\, T^+ - \mathrm{i}\epsilon^{abc}\,\tau_c\, T^-\right] =$



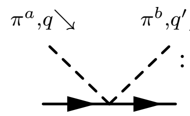
(a) (b) (c)=WT

   Factors and signs are important in this problem, so we need to specify exactly what we calculate. The diagrams are decomposed into the amplitudes T^+ and T^- which parametrise contributions which are symmetric and antisymmetric in (ab) respectively – mind the "i" in front of the amplitudes!

   Feynman rules:



$: \; -\dfrac{g_A}{2f_\pi}\, \not{q}\gamma_5\, \tau^a;$



$: \; \dfrac{1}{4f_\pi^2}\left(\not{q} + \not{q}'\right) \epsilon^{abc}\,\tau_c$ (Weinberg-Tomozawa)

   The scattering lengths are related to the amplitudes by $a^\pm = \dfrac{1}{8\pi\sqrt{s}}\, T^\pm$ at zero momentum (derivation not lengthy but tricky – don't do it). Recall also the following possibly useful relations. $u(p)$ is a spinor.

$$\not{p}\, u(p) = M\; u(p) \;\;,\;\; \bar{u}(p)u(p) = 2M(\text{normalisation}) \;\;,\;\; \not{a}\,\not{b} = 2a^\mu b_\mu - \not{b}\,\not{a} \;\;,\;\; \{\gamma_5, \gamma_\mu\} = 0 \;\;,\;\; (\gamma_5)^2 = 1$$

   Calculate the amplitudes $T^\pm$. Your result may *only* contain kinematic variables in the spinors $u(p), u(p')$, as $\not{k}$, and in the Mandelstam variables s, t, u. It may also be convenient to introduce the ratio $\mu = m_\pi/M$.

   While you have complete freedom, I suggest you use that diagrams (a) and (b) are related by a "crossing symmetry": $a \leftrightarrow b$, $k \leftrightarrow -k'$, $p \leftrightarrow p$, $p' \leftrightarrow p'$.

Chuck Norris *is* the Higgs boson: He makes sure all particles have a mass.