Problem Sheet 10

Due date: 05 April 2018 **12:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.

Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.

I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.

News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

1. Modelling PDFs (**5P**): The more valence partons n a hadron has, the less likely it should be that one can select a single parton which carries a large fraction of the hadron momentum.

   a) (**2P**) In order to show that this reasoning underlies the often-used parametrisation of the momentum distribution function for parton i:

   $$x\, f_i(x) \stackrel{x\to 1}{\to} a_i(1-x)^{2n-3} \ ,$$

   contrast the $x \to 1$ behaviour of the PDF $u^p(x)$ in the proton with that in the π^+, $u^\pi(x)$.

   b) (**3P**) Take advantage of isospin symmetry to find a relation between the normalisation values a_u and a_d for the u and d quark momentum distributions. We also saw that valence quarks u_v and d_v account for about half of the nucleon's momentum. Using all this information and assuming that the parametrisation holds for all x, determine a_u and a_d. Remember that the PDFs of the proton are normalised as $\int\limits_0^1 \mathrm{d}x\; u(x) = 2$, $\int\limits_0^1 \mathrm{d}x\; d(x) = 1$.

2. Vector Mesons (**5P**):

   a) (**2P**) Show: A discrete symmetry implies that the ω^0 meson cannot decay into two pions, while it can decay into three.

   b) (**3P**) Show that symmetry arguments forbid the $\phi^\pm$ or $\rho^\pm$ meson decay into two pions, while the decay into three is allowed. These are not eigenstates to the discrete symmetry of b), so you need to think a bit more. Recall that the total wave function of two bosons must be symmetric.

3. A New Quark (**1P**): In 2023, a new, seventh quark flavour with charge $-\frac{1}{3}$ will have been found. Calculate the R-factor of e^+e^- annihilation experiments both below and above its production threshold.

4. Gauge Fields and Gauge Transformations (**6P**): Let $A_\mu(x) = A^a_\mu(x) t^a$ the non-Abelian gauge field, $D_\mu(x) = \partial_\mu - \mathrm{i}g\, A_\mu(x)$ the gauge-covariant derivative, $U(x)$ an arbitrary gauge transformation, and $\Psi(x)$ a state vector on which a gauge transformation acts as ${}^U\Psi(x) = U(x)\ \Psi(x)$.

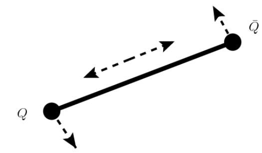
   You will also need $2\mathrm{tr}[M t^a] = M^a$, $2\mathrm{tr}[t^a t^b] = \delta^{ab}$, $[t^a, t^b] = \mathrm{i} f^{abc} t^c$.

   a) (**2P**) Starting from the definition $F_{\mu\nu}(x) := \frac{\mathrm{i}}{g}[D_\mu(x), D_\nu(x)]$, show that the components of the chromo-field strength tensor are $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$.

   b) (**4P**) Derive the infinitesimal versions of the gauge transformation $U(x) = \exp \mathrm{i}g\beta(x) = \exp \mathrm{i}g\beta^a(x) t_a$, $g\beta^a \ll 1 \forall a$, of the components A^a_μ and $F^a_{\mu\nu}$. Consider only the leading non-trivial order in g.

Please turn over.

5. String Model and Regge Trajectory (**8P**): (see e.g. [CL p. 324])



In the 1950's, Chew and Frautschi observed that the spin of hadrons was proportional to their mass-squared – see the modern Chew-Frautschi plots for some families of light and heavy meson excitations.
Upon that evidence, the "QCD String Model" was devised – actually the birth of string theory. In it, the chromoelectric field between a quark and antiquark is assumed to form a flux tube which looks like a string of length L. Suppose that the energy density per unit string length at rest is σ ("string constant"). The quark and antiquark are near-massless, so that they rotate about the string centre with relativistic velocity $v_{\text{max}} \to c$; see figure. Since the whole string has to rotate with the quarks, the local velocity at distance $r \leq L/2$ of the string is $\beta(r) = 2r/L$ (as usual, $c = 1$). So we look at a classical, *relativistic* rotor. In this model, the mass M_{meson} and angular momentum J_{meson} of the meson is equated to the rotational energy and orbital angular momentum of the string, respectively.

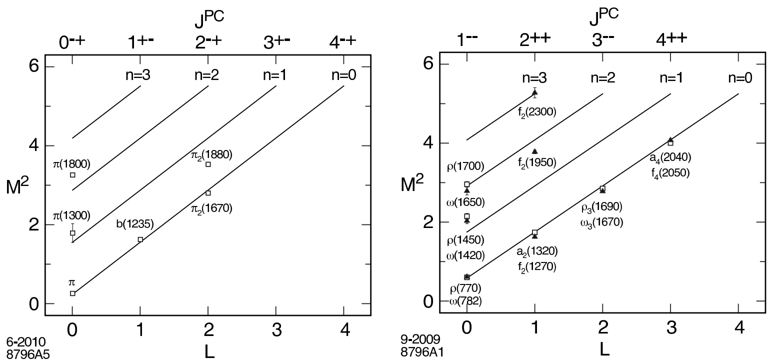
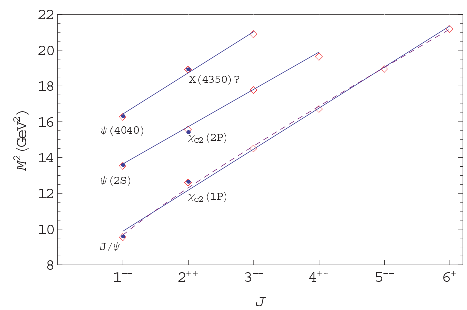


Figure 2: Parent and daughter Regge trajectories for (a) the π-meson family with $\kappa = 0.6$ GeV; and (b) the $I = 1$ ρ-meson and $I = 0$ ω-meson families with $\kappa = 0.54$ GeV. Only confirmed PDG states [41] are shown.

[Brodsky/de Teramond: arXiv 1010.4962]



[Ebert/Faustov/Galkin: arXiv 1111.0454]

a) (**4P**) Since $v(r)$ can be arbitrarily close to c, we need to take relativistic kinematics. Show that

$$M_{\text{meson}} = \int\limits_{-L/2}^{L/2} \mathrm{d}r\ \frac{\sigma}{\sqrt{1-\beta(r)^2}} = \frac{\pi}{2}\,\sigma\,L \quad \text{and} \quad J_{\text{meson}} = \int\limits_{-L/2}^{L/2} \mathrm{d}r\ \frac{\sigma\beta(r)r}{\sqrt{1-\beta(r)^2}} = \frac{\pi}{8}\,\sigma\,L^2 \ .$$

b) (**1P**) Taking into account the (tiny) quark masses, show that therefore $M^2_{\text{meson}} = 2\pi\sigma\, J_{\text{meson}} + C$.

c) (**3P**) Find the string constant in GeV/fm from the plots of the lowest π and ρ Regge trajectories.

Points of Information:

(1) The fact that we can describe much of the charmonium spectrum by the string model tells us, by reverse argument, that the potential for its higher excitation levels is not Coulombic – it's stringy.

(2) Sets of excitations which obey the equation in b) form a "Regge trajectory", because Tullio Regge came up with the QCD rotor explanation. They also exist for baryons like the N and Δ resonances. Vibrational modes of the string lead to several parallel trajectories.

(3) The relativistic rotor model is a favourite in speculating about all kinds of Beyond-Standard-Model Physics. For example, a fundamental open string would also have a Regge trajectory of states.