Problem Sheet 9 – Last for Midterm **Special Due date: Monday 19 March 2018 08:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.

Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.

I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.

News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

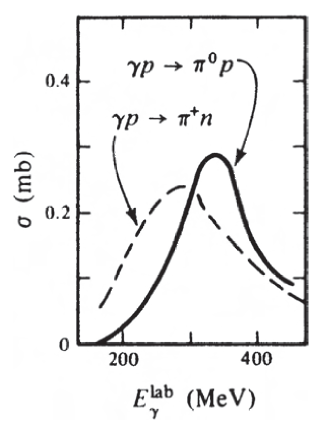
1. Forbidden States in the Constituent Quark Model (**5P**): The total wave function of a quark-antiquark system consists of its isospin, spin, orbital angular momentum and radial components:

$$\Psi = |IM_I\rangle_{\text{isospin}} \otimes |SM_s\rangle_{\text{spin}} \otimes |LM_L\rangle_{\text{orbital}} \otimes \psi_{\text{radial}}(r)$$

The total angular momentum is $\vec{J} = \vec{L} + \vec{S}$. For a system with isospin zero (i.e. both quarks have the same flavour), show that no states can be generated with the quantum numbers $J^{PC} = 0^{--}$, 0^{+-}, and 1^{-+} when L, S is 0 or 1. [Concentrate only on these; there are more forbidden states for $J > 1$.] You may find it useful to study the symmetries of the system when exchanging the position of the two quarks, and show that the parity of this state is $(-)^{L+1}$, while its charge-conjugation eigenvalue is $(-)^{L+S}$. The search for such states is a focus of the Jefferson Lab 12 GeV upgrade and of GW's Data Analysis Center DAC.

2. Isospin in Pion Photoproduction (**5P**): What looks like a resonance may just be interference.

In the process $\gamma p \to \pi N$, the $\Delta(1232)$ resonance is not seen as clearly as in πN scattering, see figure of lab frame cross section from [HG, Fig. 14.6].



   a) (**1P**) Confirm that the peak of $\gamma p \to \pi^0 p$ at about 340 MeV occurs at the expected position of the $\Delta(1232)$ resonance, while the peak of $\gamma p \to \pi^+ n$ at 290 MeV is not directly related to the Δ.

   b) (**4P**) Determine the ratio of the amplitudes $\mathcal{M}_4 := \langle I = \frac{3}{2}|S|\gamma p\rangle$ and $\mathcal{M}_2 := \langle I = \frac{1}{2}|S|\gamma p\rangle$ for two nearby energies: At the point $E_\gamma^{\text{lab}} \approx$ 300 MeV where both cross sections are identical; and at the Δ peak, where $\sigma(\gamma p \to \pi^0 p) \approx 0.3$ mb and $\sigma(\gamma p \to \pi^+ n) \approx 0.2$ mb.

3. Predicting a Cross Section (**2P**): The process $dd \to {}^4\text{He}\,\pi^0$ has been observed, but with a very small cross section. Show why you expect that, based on isospin arguments.

4. Beams, Detectors, Cross Sections (**6P**):

   a) (**2P**) ^{24}Mg is bombarded with a 10 nA α-beam; the target is quite thin (1.0 mg cm^{-2}). A detector with an opening of 2.0×10^{-3} sr records 20 protons per seconds. Assuming the scattering is isotropic, what is the cross section of ^{24}Mg(α,p)?

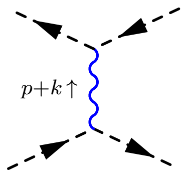
   b) (**2P**) In the CMS detector at the LHC, the hadronic calorimeters are placed outside the electromagnetic ones. The MAMI design is the reverse. Why?

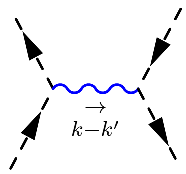
   c) (**2P**) A mixed beam contains charged pions, protons and α particles, all with the same kinetic energy of 10 MeV. Order them by their energy loss in matter and provide reasons.

Please turn over.

5. Scattering Bosonic Particle-Antiparticle Pairs (**7P**): I would love you to do Bhabha scattering $e^-e^+ \to e^-e^+$, but it involves a bit of Dirac algebra to get 8 γ-matrices down to 4. It's not particularly difficult, but it's not very entertaining either. Still, it would be a good exercise if you have time **and would be rewarded with 10 extra points** (use without proof $\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu$).

   Instead, the actual problem is: Calculate the differential cross section of scattering massless, charged spin-0 bosons $\phi^+(k)\ \phi^-(p) \to \phi^+(k')\ \phi^-(p')$ to lowest order in QED. I have labelled the bosons with their momenta. These could be the $\pi^\pm$ mesons, except without isospin. Assigning particle or antiparticle by using different arrows on the lines like for fermions, the two diagrams are

   $\mathcal{M}_a + \mathcal{M}_b =$ 

$+$ 

so that $\overline{|\mathcal{M}_a + \mathcal{M}_b|}^2 = \overline{|\mathcal{M}_a|}^2 + \overline{|\mathcal{M}_b|}^2 + 2\ \mathrm{Re}[\overline{\mathcal{M}_a^\dagger \mathcal{M}_b}]$

   Probability amplitudes in Quantum Mechanics need to be summed before one can square. At least one amplitude-squared we already calculated, and another follows for example by crossing symmetry. You already did some of this in past HWs (you can use these results with referencing). It may be best to quote the result via Mandelstam variables. Be mindful that the photon carries *different* momenta in these diagrams. Which reference frame should you use for the cross section, given that $m_\phi = 0$?

   Using a plot, compare *and interpret* the θ- or Mandelstam-t-dependence of your result, assuming all particles are massless:

   (i) to scattering two different, charged bosons ($\phi\,\Phi \to \phi\,\Phi$);
   (ii) to annihilation into a different charged-boson pair ($\phi^+\phi^- \to \Phi^+\Phi^-$);
   (iii) to $e\mu \to e\mu$;
   (iv) and to $e^+e^- \to \mu^+\mu^-$.

   State the *total cross section* in each of the 5 cases and compare; careful: some may be infinite (why?).