Problem Sheet 8

Due date: 08 Mar 2018 **12:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

1. $\Delta(1232)$ WAVE FUNCTIONS (**3P**): Construct the *combined* spin-isospin wave function of the Δ^+ and of the Δ^0 in the constituent quark model, when each is in the $M_s = \frac{1}{2}$ state.

2. DEUTERON WAVE FUNCTION (**5P**): Let's find the nucleon-nucleon spin and isospin wave functions for the part of the deuteron in which the two nucleons are in a relative s-wave. First, recall that the deuteron is a system of two identical fermions N in the isospin formalism, so its total wave function must be anti-symmetric. Second, recall that there is only one deuteron state, and not three facets with different charges. Third, the deuteron is predominantly s-wave.

   a) (**3P**): Show now: The deuteron must be a $J = 1$ state, and its parity must be positive. This proves the assertion made in the CTP section that the deuteron is a pseudo-vector.

   b) (**2P**): Write down the spin-isospin wave function of the deuteron, for spin-magnetic quantum number $M = 0$.

3. DECAY OF A MASSIVE VECTOR PARTICLE (**6P**): The following could serve as a first shot to describe the decay of vector mesons like the $\omega^0(782)$ ($J^{PC} = 1^{--}$), or of the Z boson of the electroweak theory. We assume either can be described by the Lagrangean of a real (i.e. charge-neutral) Lorentz-vector field B^μ, see previous HWs.

   You can use the following without proof: A vector field has three spin states (polarisation vectors) $\vec{\epsilon}^{(M)}$ with $M = \pm 1, 0$. For a B particle at rest, they span all of space, i.e.

   $$\sum_{M=0,\pm 1} \vec{\epsilon}^{*(M)} \otimes \vec{\epsilon}^{(M)} = 1, \text{ or in Einstein's Summation convention } \sum_{M=0,\pm 1} \epsilon_i^{*(M)} \epsilon_j^{(M)} = \delta_{ij},$$

   or in the relativistic version (not at rest): $\displaystyle\sum_{M=0,\pm 1} \epsilon_\mu^{*(M)} \epsilon_\nu^{(M)} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}$.

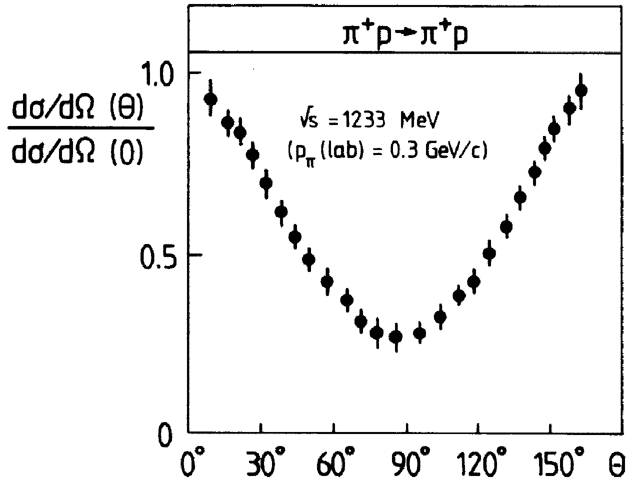
   You already showed that the coupling of the vector boson to a fermionic current is the same as for electron-photon coupling, so we can take over that Feynman rule. Let's denote the coupling constant by g (identical to e in scalar QED).

   Calculate now the width of the decay $B \to e^+ e^-$ for massless electrons. The result is $\Gamma = \dfrac{g^2 M}{12\pi}$.

4. ISOSPIN BREAKING BY ELECTROMAGNETISM (**2P**): The nuclei ^{3}He and ^{3}H (also called the triton) have nearly the same total binding energies: $B(^3\text{He}) = 7.7$ MeV, $B(^3\text{H}) = 8.5$ MeV. Attributing the difference to electrostatic repulsion in ^{3}He, estimate the mean distance between its two protons using classical arguments.

Please turn over.

5. Spin of the $\Delta(1232)$ (**5P**):



The angular distribution of the differential cross section of $\pi^+ p \to \pi^+ p$ around $\sqrt{s} = 1232$ MeV (see figure) is parametrised by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta = 0)\left[A + B\ \cos\theta + C\ \cos^2\theta\right]$$
$$= |g(\theta, E)|^2 + |h(\theta, E)|^2\ .$$

Its decomposition into partial wave amplitudes is:

$$g(\theta, E) = \sum_{l\geq 0}\left[(l+1)\, a_{l+}(E) + l\, a_{l-}(E)\right] P_l(\cos\theta)$$

with $h(\theta, E) = \sin\theta \sum\limits_{l\geq 0}\left[a_{l+}(E) - a_{l-}(E)\right] P_l'(\cos\theta)$.

P_l is the lth Legendre Polynomial, and $P_l' = \frac{\mathrm{d}P_l(x)}{\mathrm{d}x}$. The coefficients $a_{l\pm}(E)$ are the partial-wave amplitudes to total angular momentum $l \pm \frac{1}{2}$ and can be assumed to be real.

a) (**1P**) Find A, B, C from the plot by rough estimates at $\theta = 0, \frac{\pi}{2}$ and considering the overall shape.

b) (**4P**) Match the two parametrisations (for S- and P-waves only). Show that the $\Delta(1232)$ resonance indeed has $l = 1$ and $J = \frac{3}{2}$ – assuming there is no fine-tuning between unrelated partial waves. This vindicates the hand-waving argument based on the Breit-Wigner formula.