

## Problem Sheet 7

Due date: 01 March 2018 **12:00**

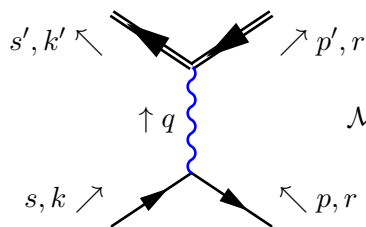
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**Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.**

*I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.*

News and .pdf-files of Problems also at [home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html](http://home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html).

1.  $e^+e^- \rightarrow \mu^+\mu^-$ , PART II (**8P**): We continue the calculation of electron-positron annihilation into muons. In the last HW, you derived its matrix element for an electron (spin  $s$ , 4-momentum  $k$ ) and positron (spin  $r$ ; 4-momentum  $p$ ) converted into a muon and anti-muon of spins  $s'$  and  $r'$  and momenta  $k'$  and  $p'$  via a virtual photon of 4-momentum  $q$  as:



$$\mathcal{M}_{sr;s'r'}(k, p; k', p') = -Ze^2 \bar{v}_r(p) \gamma^\mu u_s(k) \frac{1}{q^2 = (k+p)^2} \bar{U}_{s'}(k') \gamma_\mu V_{r'}(p') .$$

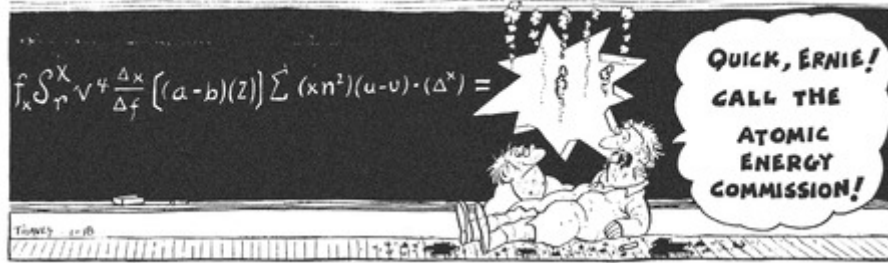
Upper-case spinors denote muons/anti-muons, and lower-case spinors  $e^\pm$ . Now finish the calculation:

- (**4P**) Calculate the spin-averaged matrix-element squared for massless particles, recalling our normalisations  $\sum_s u_s(p) \bar{u}_s(p) = p \cdot \gamma + m$  and  $\sum_s v_s(p) \bar{v}_s(p) = p \cdot \gamma - m$ .
  - (**3P**) Rewrite it into Mandelstam variables and show the crossing symmetry to  $e\mu \rightarrow e\mu$ . The result is proportional to  $\frac{t^2 + u^2}{s^2}$ .
  - (**1P**) Derive the differential and total cross section in the cm frame (result given in the lecture).
2.  $\beta$  DECAY OF NUCLEI IN THE LIQUID-DROP MODEL (**4P**): Write a *short* essay which compares the  $\beta$  decay of an odd- $A$  nucleus against that of an even- $A$  nucleus. Provide qualitative arguments on the difference in typical decay rates or stability of the respective nucleus against that decay. You should base your arguments on the Bethe-Weizsäcker formula and include instructive sketches and examples. Yes, you may get inspired by a good textbook. This is a “Reading Assignment”.
3. BETHE-WEIZSÄCKER FORMULA (**4P**): Let’s have a bit more qualitative fun. These problems are independent of each other.
- (**1P**) Compare the (classical) electromagnetic self-energy of a uniform sphere with radius  $R = 1.21\text{fm} \times A^{1/3}$  and total charge  $Ze$  to the value of the parameter  $a_C$  in the Bethe-Weizsäcker formula.
  - (**3P**) The total binding energy of  ${}^4\text{He}$  is 28.3 MeV. Show that  $\alpha$  decay becomes energetically allowed for nuclei around the bottom of the valley of stability when  $A \gtrsim 150$ .

**Please turn over.**

4. VIRTUAL PARTICLE CLOUDS AND CHARGE RADII (4P): By the uncertainty principle, every particle is surrounded by a cloud of virtual particles. Charged hadrons can emit virtual photons, and these virtual photons in turn generate virtual electron-positron pairs. So hadrons are also surrounded by  $e^+e^-$  pairs. Draw at least two Feynman diagrams which describe their coupling to the photon exchanged in electron scattering off hadrons. By estimating the size of this contribution to the hadron charge radius, show that such effects are much smaller than that from virtual mesons.

### Frank and Ernest



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