Problem Sheet 6

Due date: 22 February 2018 **12:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

1. FILL IN THE DETAILS: LEPTON TENSOR (**4P**): In the lecture, the lepton tensor was defined as

$$e^2\; L^{\mu\nu} = \frac{1}{2s+1}\sum_{s,s'} j^{\mu}_{s,s'}(k,k')j^{\nu\dagger}_{s,s'}(k,k')$$

and calculated for massless leptons. Derive now the result for the case $m \neq 0$ ***and show by explicit calculation that*** $q_\mu L^{\mu\nu} = 0$ ***for this case***, where $q_\mu = k_\mu - k'_\mu$ as usual in our standard kinematics. You will need this below. Work with the notes and [HM, sects. 6.3/4].

2. FILL IN THE DETAILS: MOTT CROSS SECTION (**5P**): For electron scattering on a massive point-particle without spin, the lecture jumped from the averaged and squared matrix element straight to the lab cross section. So, fill in the details of the following (take $L_{\mu\nu}$ as given):

$$|\bar{\mathcal{M}}|^2 = \frac{(Ze^2)^2}{q^4}\; L_{\mu\nu}\;(p+p')^\mu\;(p+p')^\nu \quad\Longrightarrow\quad \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\mathrm{lab}} = \left(\frac{Z\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}\quad .$$

Hints: It may be useful to use $q_\mu L^{\mu\nu} = 0$ (because Feynman rules for on-shell particles must obey current conservation, see lecture) and/or an algebraïc manipulation programme like Mathematica. Do not specify a frame to calculate $|\overline{\mathcal{M}}|^2$; wait until you really look at the cross section.

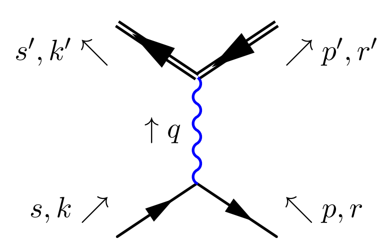
3. SCATTERING OF SPIN-0 PARTICLES (**7P**): Let's consider electromagnetic scattering between two different massless point-particles with spin zero and charge e ("spinless" version of $e\mu \to e\mu$). Do you expect angles under which scattering is forbidden? Calculate the differential cross section in the cm frame, at lowest non-vanishing order.

Fail-Safe Point: An intermediate step is $|\bar{\mathcal{M}}|^2 = \dfrac{e^4}{q^4}\;[(p+p')\cdot(k+k')]^2$, in our "standard" kinematics.

4. INELASTIC SCATTERING (**2P**): Its most general hadron tensor was given in the lecture.
Show that $W_1(Q^2,x) = 0$ for a spin-zero target. Since targets without spin cannot directly couple to magnetic fields, that's pretty good evidence that W_1 parametrises magnetic interactions.

5. $e^+e^- \to \mu^+\mu^-$, PART I (**2P**):
Write down the Feynman diagram and matrix element describing the annihilation of an electron (spin s, 4-momentum k) and positron (spin r; 4-momentum p) into a muon and anti-muon of spins s' and r', respectively, at lowest non-vanishing order.
Do not do the calculation. – We do that in the next HW.

Follow the conventions used in this Feynman diagram (time goes up)! Notice that momentum flow and arrow indication for particle/antiparticle can be different.



A neutron walks into a bar and asks "How much for a beer?"
The bartender says, "For you? No charge."