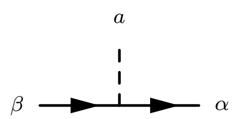
Problem Sheet 5

Due date: 15 February 2018 **12:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

1. Feynman Rule (**up to 5P more**): **Problem sounds familiar?: Yes, it's the same as last week. Now, you can recover lost points.** Derive the Feynman rule of the interaction between a Pauli-spinor ψ (2-dimensional vector in spin space) and a real scalar field π via the three Pauli matrices σ^a:

$$\mathcal{L}_{\pi N} = -\frac{g_A}{2 f_\pi}\psi^\dagger\left(\vec{\sigma}\cdot(\vec{\partial}\pi)\right)\psi \quad\Longrightarrow$$



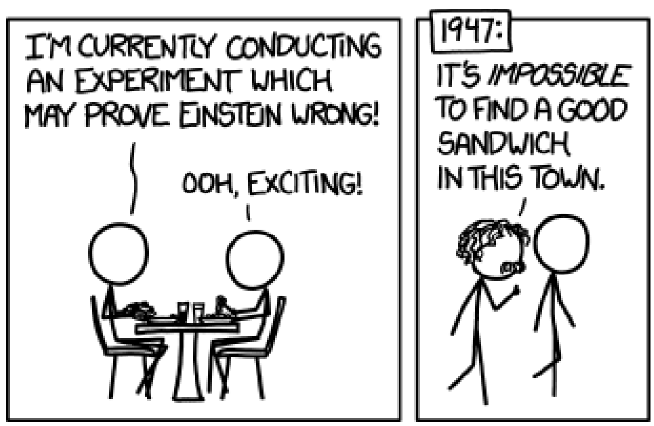
The spin of the incoming ψ is β, that of the outgoing one is α. You need to label the momentum of at least one of the particles as well. Make sure that your rule matches the labelling conventions of this Feynman diagram and that *all* indices appear in your Feynman rule!

This is a simplified version of the lowest-order interaction between a non-relativistic nucleon (i.e. no anti-nucleon) and the pion field. One finds $g_A = 1.267$ and $f_\pi = 92.21$ MeV [PDG].

2. From Counts to Cross Sections (**7P**): In an experiment at nuclear/hadronic energy scales, you count 96 particles you want in a detector which covers $[92.704 \pm 0.033]\%$ of all solid angles. [You can measure the opening with very high accuracy, so you do not associate an error bar with that. "4π detectors" cover scattering under all angles.] Let's assume the cross section and all corrections you need are isotropic (i.e. do not depend on the scattering angle θ). You determined your total experimental efficiency as $\epsilon = [4.5 \pm 0.3] \times 10^{-6}$. You cannot record events which are less than 7s apart ("dead time"). The luminosity was $[3.3 \pm 0.2]$kHz/pbarn, and you had 123 hours and 3 minutes of high-quality beamtime. All statistical errors are uncorrelated and normal-distributed (you hope).

   a) (**1P**) Do you need to correct for "dead-time"?
   b) (**4P**) How big was your total cross section, and how big is the combined error?
   c) (**2P**) Argue if the process is strong, weak or electromagnetic.

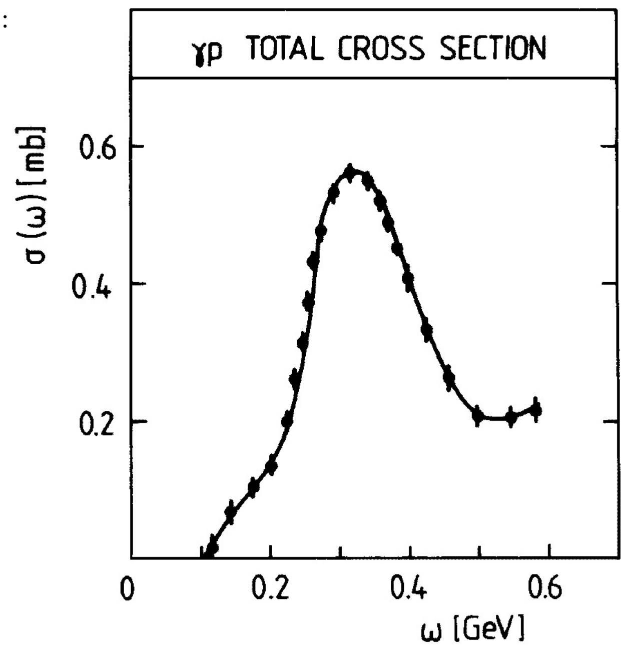
3. Lorentz Invariants (**3P**): In the lecture, I stated that the Lorentz invariant luminosity is for a theorist $L = 4\sqrt{(p\cdot k)^2 - m^2 M^2}$. Show that this reduces to $4|\vec{k}_{\text{lab}}|\,M$ in the lab frame, and $4|\vec{k}_{\text{cm}}|\,\sqrt{s}$ in the cm frame. Also, what are the units of this expression, and do you expect these?



Please turn over.

4. The $\Delta(1232)$ Resonance in γp Scattering (**5P**):

   The total cross section of photon scattering on protons shows for $\omega < 600$ MeV a resonance with spin $J = 3/2$ (plot in *lab* frame).

   a) (**4P**) Take the figure to approximate the cross section around the resonance by the Breit-Wigner form and extract the position of the pole of the scattering matrix in the complex-energy plane, i.e. the invariant mass M_Δ and decay width Γ. In the cm frame, let's take a quasi-nonrelativistic form, with $E_{\text{cm}}^{\text{total}} = \sqrt{s}$:
   $$\sigma \propto \frac{1}{|\vec{k}_{\text{cm}}|^2} \frac{1}{(E_{\text{cm}}^{\text{total}} - M_\Delta)^2 + \Gamma^2/4}$$

   b) (**1P**) Why is M_Δ *not* just given by the peak position in the plot, plus the proton mass?

5. Form Factor for a Uniformly Charged Sphere (**5P**): Find the form factor of a homogeneously charged sphere with total charge Ze, i.e. $\rho =$const. for $r < R$, zero otherwise. This is useful for heavy nuclei. Show that this form factor has a zero at $qR \approx 4.5$, and that the relation between root-mean-square radius and sphere size is $\langle r^2 \rangle = \frac{3}{5} R^2$. Calculate also the relation between R, q and the second zero of the form factor. Always remember: using an algebraïc manipulation programme bears no penalty except that you save time.