Problem Sheet 4

Due date: 08 February 2018 **12:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

1. Feynman Rule (**5P**): Derive the Feynman rule of the interaction between a Pauli-spinor ψ (2-dimensional vector in spin space) and a real scalar field π via the three Pauli matrices σ^a:

$$\mathcal{L}_{\pi N} = -\frac{g_A}{2f_\pi}\psi^\dagger\left(\vec{\sigma}\cdot\vec{\partial}\pi\right)\psi \implies$$

The spin of the incoming ψ is β, that of the outgoing one is α. You need to label the momentum of at least one of the particles as well. Make sure that your rule matches the labelling conventions of this Feynman diagram!

This is a simplified version of the lowest-order interaction between a non-relativistic nucleon (i.e. no anti-nucleon) and the pion field. One finds $g_A = 1.267$ and $f_\pi = 92.21$ MeV [PDG].

2. Standard Toy Model of Low-Energy Nuclear Physics (**5P**): At kinetic energies much smaller than the particle mass, we can resort to a non-relativistic theory. We thus explore the complex scalar field $\Phi(x)$ of a particle with mass M and charge Ze, using the "cooking recipes" of the lecture. The Lagrangean is

$$\mathcal{L} = \Phi^\dagger[\mathrm{i}\partial_0 + \frac{\vec{D}^2}{2M}]\Phi \;-\; C\,(\Phi^\dagger\Phi)^2$$

where $\vec{D} = \vec{\partial} - \mathrm{i}Ze\vec{A}$ is the gauge-covariant derivative.

   a) (**2P**) Derive the propagator for energy p_0 and momentum $\vec{p}$. It is *not* relativistic: $\dfrac{\mathrm{i}}{p_0 - \frac{\vec{p}^2}{2m} + \mathrm{i}\epsilon}$.
   b) (**2P**) Show that such particles can only propagate forward in time. (Recall the discussion of causality and Fourier transforms in Math. Methods?)
   c) (**3P**) Derive the Feynman rules for all three interactions, with their corresponding diagrams.

3. Positronium (**5P**) is the hydrogen-like bound state between electron and positron, where the spins can couple to total spin 0 (para) or 1 (ortho). This state has a finite lifetime since the probability amplitude for both particles to overlap is nonzero when they are in a relative S-wave.

   a) (**2P**) Using a discrete symmetries, consider: For the decay into photons only, at least how many photons are generated from para-positronium? How many from ortho-positronium?
   b) (**1P**) Estimate the ratio of the two life times.
   c) (**2P**) Using a discrete symmetries, show that the positronium decay into $\pi^0\pi^0$ is forbidden. [Yes, it's also kinematically forbidden, but let's pretend that we live in a world where $m_\pi < m_e$.]

Please turn over.

4. Electron Scattering on Deuterons (**5P**): In Spring 2014, the A1 collaboration at MAMI conducted an experiment on $d(e,e')d$ scattering to determine the deuteron form factor. The cross section for the process is on the 10% level predicted as (see next lectures for details):

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\mathrm{lab}} = \left(\frac{\alpha}{2E\sin^2\frac{\theta}{2}}\right)^2 \cos^2\frac{\theta}{2}$$

According to the MAMI beam schedule (online), a beam of 20 μA and 1 cm^2 cross section is focused on a liquid-deuterium target with 1.0×10^{23} deuterons per cm^2 of beam cross section. (Yes, they constructed the target to give a nice and round number!) The experiment is run at many angles and energies, but we pick a 200 MeV beam and 100° scattering angle. The scattered electrons are detected with one of the A1 spectrometers. Its opening is 28 msr (opening must be very small because the momentum resolution must be very good). Let's say the detector system has an efficiency of 50%. Confirm that you expect a cross section of about 0.15 μb/sr; determine the luminosity in MHz/μb; find the event rate in Hz, i.e. the number of events counted per second; and finally calculate (estimate) the runtime to achieve a measurement with a statistical accuracy of 1%.