## Problem Sheet 3

Due date: 01 February 2018 **12:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.

**Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.**

*I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.*

News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

1. Going Underground (**3P**): A look at the plot of the Bethe formula also explains why cosmic-ray muons are still observed in mines that are more than 1 km underground. Roughly estimate the minimum initial energy of these muons when their detected energy is at least 1 GeV. Why are no cosmic-ray protons or pions observed in these underground laboratories?

2. Bragg Curve (**5P**): The Bethe-Bloch formula provides the energy loss $-\mathrm{d}E/\mathrm{d}x$ as function of particle momentum. Based on it and its graphical representation, sketch the typical energy loss as function of distance travelled in the material, for a particle with $\beta\gamma \approx 1$ and $\beta\gamma \approx 100$. A qualitative plot suffices - do *not* derive formulae! Discuss how the maximum in the first case can be utilised in cancer treatment. Do you expect a similar curve for photons (why/not)?

3. Energy Loss by Compton Scattering (**4P**): A photon of energy $E$ is scattered on a particle of mass $M$ under angle $\theta$, in the rest frame of the particle. Show that its energy after the event is

$$E' \leq \frac{E}{1 + \frac{E}{M}(1 - \cos\theta)} \quad \text{(equality for elastic scattering).}$$

   Provide a *qualitative* argument that energy loss by Compton scattering is, averaged over angles, large in the window in which this process dominates over bremsstrahlung or pair-production. You can for example assume that the cross section is very roughly angle-independent.

   **Note:** This justifies attributing an "attenuation index" to the process, although the initial photon does not disappear. And it demonstrates that scattering on electrons is important for attenuation, while scattering on nuclei is not.

4. Threshold Čerenkov Detector (**2P**): Tune the optimal index of refraction in a Čerenkov counter such that it separates pions from kaons in a beam of particles with momentum 20 GeV.

   **Up To 5 Bonus Points:** The index of refraction can be tuned quite finely by varying the gas pressure. What is the gas pressure necessary to achieve that differentiation between pion and kaon for Nitrogen as filling gas, assuming that $N_2$ is an ideal gas?

**Please turn over.**

5. MASSIVE VECTOR PARTICLES (**6P**): The Lagrangean of a massive real spin-1 field $B^\mu$ looks like that for the photon, but with a mass term. This could for example be one of the vector mesons $\rho$, $\omega$ ($M \approx 770$ MeV each; $J^{PC} = 1^{--}$), $\phi$ ($M \approx 1020$ MeV), or the $W/Z$ weak exchange boson ($M = 80/90$ GeV). Another term describes coupling to an external current $j^\mu$:

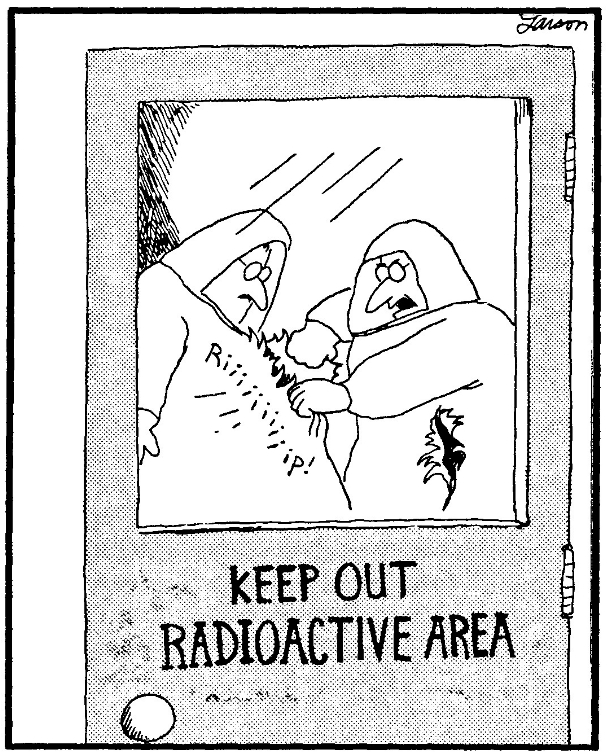
$$\mathcal{L}_{\text{massive spin-1}} = -\frac{1}{4}\ (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \frac{M^2}{2} B_\mu B^\mu - g\, B_\mu j^\mu$$

a) (**2P**) Show: The equation of motion is $\partial_\mu(\partial^\mu B^\nu - \partial^\nu B^\mu) + M^2\ B^\nu = g\, j^\nu$ (PROCA-EQUATION).

b) (**1P**) Show: $j^\mu$ is conserved if the vector field obeys $\partial \cdot B = 0$ (cf. LORENTZ gauge condition).

c) (**2P**) Show: The propagator is $\mathrm{i}D_F^{\mu\nu}(k) = \dfrac{-\mathrm{i}}{k^2 - M^2 + \mathrm{i}\epsilon}\left[g^{\mu\nu} - \dfrac{k^\mu k^\nu}{M^2}\right]$.

d) (**1P**) Show: The propagator obeys current conservation on-shell, i.e. $k_\mu D_F^{\mu\nu}(k) = 0$ at $k^2 = M^2$.




"So, Foster! That's how you want it, huh?...Then take THIS!"