Problem Sheet 2

Due date: 25 January 2018 **12:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

1. Why Always Forward? (**3P**): Radiation off ultra-relativistically moving particles is strongly peaked around the direction of the velocity. Let's see how that emerges in a simplified picture. An electron travels in the x-direction at speed $\beta \to 1$ relative to an observer and emits a photon along its y-axis. Show that the photon appears for the observer at an angle $\theta \approx 1/\gamma$ relative to the electron's motion in his frame. Show also that all photons emitted in the forward hemisphere in the moving frame appear inside a cone of solid angle π/γ^2 in the lab frame.

   Hints: Yes, I know that many of you are familiar with that problem – so review it! The following may come in handy:

$$\text{photon position 4-vector in } e^-\text{-rest frame at time } t_{e^-}\text{: } x^\mu_{e^-}(t_{e^-}) = \begin{pmatrix} ct_{e^-} \\ x_{e^-} = 0 \\ y_{e^-} = ct_{e^-} \\ z_{e^-} = 0 \end{pmatrix}$$

$$\sin\theta_{\text{lab}} = \frac{y_{\text{lab}}}{x_{\text{lab}}} \quad .$$

2. Relativistic Kinematics (**5P**): For scattering a massless particle (photon) of energy E on a mass M, we consider the transformation of variables between the cm and lab frames.

   a) (**1P**) Show that $E_{\text{lab}}\, M = \sqrt{s}\, E_{\text{cm}}$, where s is one of the Mandelstam variables.

   b) (**3P**) Show that the scattering angle transforms as

$$\cos\theta_{\text{cm}} = \frac{\cos\theta_{\text{lab}} - \beta}{1 - \beta\cos\theta_{\text{lab}}}$$

   where $\beta = E_{\text{lab}}/(E_{\text{lab}} + M)$ is the relative velocity between the cm and lab frames.

   c) (**1P**) When the photon is scattered without directional preference in the cm frame, does that also hold in the lab frame?

3. Sorting Out Reactions (**6P**): Here are some seemingly very similar processes. Some, however, are ruled out in the Standard Model, one is predicted to be quite improbable, and only one proceeds at a decent rate. Identify each, with reasons, specify the fundamental interaction through which it proceeds, and estimate the cross section ratio between the improbable and probable process. If there is a minimum threshold energy below which the process cannot occur, find it.

   (a) $\Lambda + \pi^0 \to \text{n} + K^0$
   (b) $\Lambda + \pi^0 \to \text{n} + \bar{K}^0$
   (c) $\Lambda + \pi^0 \to \text{n} + \pi^0 + \nu_e$
   (d) $\Lambda + \pi^0 \to \text{n} + \pi^0$
   (e) $\Lambda + \pi^0 \to K^0 + \pi^0 + \pi^0$
   (f) $\Lambda + \pi^0 \to \text{n} + \pi^0 + \pi^+$

   Hints: You do not need to consider T, C or P symmetries. Lepton-/baryon-number and charge conservation may come in handy. It may help to decompose the particles into their quark contents, so you need some source for that.

Please turn over.

4. ONE MORE DIRAC RELATION (**1P**) Show $\gamma_\mu \not{p} \gamma^\mu = -2\not{p}$.

5. A HUGE ELECTRON ACCELERATOR (**2P**): CERN's LEP-II electron-positron collider has a 2% energy loss per orbit due to synchrotron radiation (ring circumference 27 km, beam energy $\approx$ 100 GeV). Let's assume that this sets the limit of what we just can tolerate. What is the maximum beam energy we could get if we increased the tunnel to span Earth at the Equator? The formula for energy-loss per orbit for synchrotron radiation is on the slides.

6. STRONG OR NOT? (**1P**) In the lecture, a simple argument for a strong decay was that its time scale is set by the time it takes for information to spread through the particle. Is the life time of the Ξ baryon indicative of a strong process?

7. ELECTRONS IN WATER (**2P**): An electron with initial energy 2 GeV traverses 10 cm of water. Look up its radiation length and calculate its final energy.