Problem Sheet 1

Due date: Thursday 18 January 2018 12:00

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-18I/nupa-18I.html.

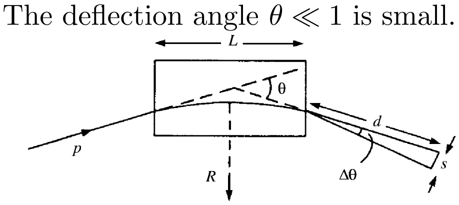
You may have to look up values in [PDG]. – Get used to it!

1. Resolution (**2P**): An experiment aims to resolve the Kaon-substructure. Estimate the minimum probe momentum necessary. **Hints:** Optical resolution, uncertainty principle, blahblahblah…

2. Natural System of Units (**3P**): Translate the following quantities (given in natural units and the Heaviside-Lorentz system) into SI units.

   a) (**1P**) Typical speed in a neutron gas with temperature 300K.
   b) (**1P**) The mass of the η meson.
   c) (**1P**) The flux of primary cosmic rays, averaged over the Earth's surface, is about 1 $\text{cm}^{-2}\text{s}^{-1}$, and their average kinetic energy is 3 GeV. Show that the power transferred is about 2 gigawatt.

3. Momentum Selector (**5P**): A $L = 1.5$ m long bending magnet produces a field of $B = 1.2$ T. You use it to deflect protons through a $s =$ 2mm wide collimator slit. The deflection angle $\theta \ll 1$ is small.



   a) (**2P**) Show by converting between SI and natural units that the curvature radius R relates to the particle momentum p via $\frac{p}{[\text{GeV}]} \approx 0.3 \frac{R}{[\text{m}]} \frac{B}{[\text{T}]}$.
   b) (**2P**) How far from the magnet should you put the slit in order to select only protons with a momentum of 49 to 51 GeV?
   c) (**1P**) Now vary the location of the same collimator on the arc of a circle with fixed distance from the centre of the magnet. Show that the relative momentum resolution scales as $\frac{\Delta p}{p} \propto p$.

4. Dirac Matrices (**5P**): In anticipation of the QFT reminder, prove the following results for the traces over Dirac matrices, using the Dirac algebra $\{\gamma^\mu, \gamma^\nu\} = 2\, g^{\mu\nu}$, symmetry properties of the trace, $\gamma_5^2 = 1$ and $\{\gamma_5, \gamma^\mu\} = 0$, where $\gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$ is the chirality operator (see [HH QM, chap. 10]):

   a) (**1P**) $\text{tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}$
   b) (**2P**) $\text{tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4\,[g^{\mu\nu}\, g^{\rho\sigma} - g^{\mu\rho}\, g^{\nu\sigma} + g^{\mu\sigma}\, g^{\nu\rho}]$
   c) (**2P**) The trace over an odd number of γ matrices is zero.

   Hint: As usual, you are allowed to consult a good book.

5. Muon Decay (**2P**): The collider at the Fermi National Accelerator Laboratory (Fermilab/FNAL) close to Chicago produces muons with an energy of 100 GeV. A muon decays in its rest frame into an electron, a neutrino and an anti-neutrino. What is its life-time in the lab frame? How far can it typically travel before the decay?

6. Synchrotron Radiation (**3P**): Discuss emission direction and polarisation of synchrotron radiation from ultra-relativistic particles, relative to the direction of acceleration and velocity, respectively.

Chuck Norris smashes elementary particles in halves – for breakfast.