# Problem Sheet 9

Due date: 29 March 2017 **16:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
**Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.**
*I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.*
News and .pdf-files of Problems also at http://home.gwu.edu/~hgrie/lectures/edyn17/edyn17.html.

1. Larmor's Formula and Modern High-Energy Physics (**6P**): Will the frontiers of high-energy physics in future be pushed by linear accellerators or by storage rings?

   The relativistic version of Larmor's formula can be re-written as (Liénard 1898):

   $$P = \frac{2q^2}{3c}\,\gamma^6\left[\left(\dot{\vec{\beta}}\right)^2 - \left(\vec{\beta}\times\dot{\vec{\beta}}\right)^2\right]$$

   a) (**3P**) Discuss radiation loss from ultra-relativistic particles in linear accelerators vs. storage rings when the magnitude of the *total* force exerted on the particle is the same in both cases.

      **Hint**: Recall that the momentum is $\vec{p} = m\gamma(v)\,\vec{v}$. Is $|\vec{v}|$ constant for a storage ring/for a linear accelerator? Is therefore the acceleration the same in both cases, if the force exerted is the same?

   b) (**3P**) The Large Electron-Prositron-Storage Ring LEP-2 at CERN is with a circumference of 27 km likely to be the largest storage ring ever built: Its state-of-the-art superconducting magnets store electrons and positrons with an energy of up to 90 GeV, the injectors and accelerating cavities barely keeping up with radiation loss. Which fraction of the electron energy is per cycle lost to radiation?

      **Note:** At LEP-2, one discovered the $W$ and $Z$ bosons, i.e. the missing particles which mediate the unified force of electro-magnetic and weak interactions, and pinned down the "Standard Model of Fundamental Interactions" (Nobel 1979, 1984, 1992, 1999, 2004, 2008). With the LHC, we hope to find the Higgs-particle which gives mass to all other particles, and see beyond the Standard Model.

2. The Falling Electron (**9P**): An electron is initially at rest and then falls for a few metres without friction, attracted by homogeneous gravitational field of the Earth. We consider its velocity to be non-relativistic at all times. The observer stands a few dozen metres away. You will see in a) that the wavelength of the radiation is much larger than the typical distance the electron falls.

   a) (**4P**) How does the radiation emitted depend on: height of fall; charge and mass of the electron; speed of light; acceleration factor; distance to the observer; time $T_{\mathrm{max}}$ over which the particle falls (non-relativistically!); weather conditions; other parameters?

      Based on that, use dimensional analysis to estimate the typical wavelength of the radiation. Give a number with a sensible length-scale attached. Under which conditions can one use the long-wavelength approximation?

   b) (**3P**) Determine the time-dependent electric field. In which directions lie the radiation maxima, at a given instant in time. Find also (by *thinking*, not by calculating) direction and value of the minima of the angular characteristics.

   c) (**2P**) After the particle has fallen some distance $h$, compare the energy radiated away to the gravitational potential energy lost.

**Please turn over.**

3. RELATIVISTICALLY OSCILLATING CHARGE (**15P**): A charge $q$ with mass $m$ oscillates harmonically about the origin on the $z$-axis, $z(t) = l\cos\omega t$, with possibly relativistic velocity.

   a) (**3P**) Before you start, discuss which radiation characteristics you would expect, based on your knowledge of the radiation characteristics of non-relativistically and relativistically oscillating dipoles. It may be useful to plot or discuss the time-dependent radiation characteristics for $\omega t = 0; \pi/2; \pi; 3\pi/2$.

   b) (**3P**) Show that the radiated power per unit solid angle is with $\beta := l\omega/c$:

   $$\frac{\mathrm{d}P(t)}{\mathrm{d}\Omega} = \Gamma\ \frac{\sin^2\theta\ \cos^2\omega t}{(1+\beta\cos\theta\sin\omega t)^5}$$

   Determine $\Gamma(\beta, q, l)$. What is the physical meaning of the symbol $\beta$?

   c) (**3P**) Determine the time-averaged power radiated per unit solid angle. You may have to perform a contour integration about the unit circle, after a change of variables $\mathrm{e}^{\mathrm{i}\omega t} = z$. You actually did it already on the last Mathematical Methods HW sheet:

   $$\frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\alpha\ \frac{\cos^2\alpha}{(1+a\sin\alpha)^5} = \frac{1}{8}\ \frac{4+a^2}{(1-a^2)^{\frac{7}{2}}}\ \text{, where in Physics } \alpha = \omega t \text{ and } a = \frac{l\omega}{c}\cos\theta\ ,\ |a| < 1\ .$$

   d) (**6P**) Compare the time-averaged radiation characteristics for the non-relativistic case to the relativistic one, and to Hertz' dipole. Include sketches of the characteristics at selected values of $\beta$. Also, plot the time-dependent characteristics at the times given in a).