Problem Sheet 8 – Last Relevant for Midterm **Special due: Mon 20 March 2017 *09:00***

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at http://home.gwu.edu/~hgrie/lectures/edyn17/edyn17.html.

1. Radiation Field of a Rotating Sphere (**12P**): Consider a spherical shell of radius R with its centre located at the origin. The shell is infinitely thin, and the total charge Q is distributed uniformly across the shell. The sphere rotates non-relativistically around the z-axis with a given, time-dependent frequency $\omega(t)$. A point charge $-Q$ is located at the centre of the sphere.

   a) (**1P**) Show that the current density $\vec{j}(t,\vec{r}) = \rho(r)\,\vec{\omega}(t)\times\vec{r}$ has a vanishing divergence. Show from this that the radiation cannot be electric in origin.

   b) (**2P**) Calculate the time-dependent vector potential of this distribution at large distances from the sphere, taking into account retardation at leading non-vanishing order:

   $$\vec{A}(t,\vec{r}) = \frac{QR^2\dot{\omega}(t-\frac{r}{c})}{3rc^2}\,\vec{\mathrm{e}}_z\times\vec{\mathrm{e}}_r$$

   Hint: You may have to prove $\int \mathrm{d}\Omega'\,(\vec{\mathrm{e}}_r\cdot\vec{\mathrm{e}}_{r'})\,\vec{\mathrm{e}}_z\times\vec{\mathrm{e}}_{r'} = \frac{4\pi}{3}\,\vec{\mathrm{e}}_z\times\vec{\mathrm{e}}_r$; see Mathematical Methods. It might be beneficial to use the radiation formula derived in the last HW.

   c) (**1P**) Calculate the time-dependent electric or magnetic field.

   Fail-safe point: With Γ a constant involving Q, R, c, a result can be cast into the form

   $$\vec{E}(t,\vec{r}) = \frac{\Gamma}{r}\,\ddot{\omega}(t-\frac{r}{c})\,\vec{\mathrm{e}}_r\times\vec{\mathrm{e}}_z\quad.$$

   d) (**2P**) Discuss the polarisation of the radiation field emitted in the following directions:
      (i) in (very close to) the xy-plane;
      (ii) in (very close to) the positive or negative z-direction.

   e) (**2P**) Determine and sketch the angular distribution of the emitted radiation. Compare to radiation from a Hertz dipole.

   f) (**1P**) Evaluate the total radiated power at time t.

   g) (**2P**) At which frequency does the total radiated power have its maximum? Can one answer the question without knowing more about $\omega(t)$?

   h) (**1P**) Discuss and interpret your result for constant angular velocity.

Please turn over.

2. Radiating Charges in Front of a Wall (**12P**): Two charges of equal but opposite magnitude are put in front of an infinitely long, perfectly conducting wall which fills all space for $x < 0$. They are infinitesimally apart and harmonically oscillate against each other with non-relativistic velocities on a straight line parallel to the z-axis, producing a (time-dependent) dipole moment $\vec{d}$. The wave-length of the emitted radiation is *not necessarily* large against the distance $\vec{a} = a\vec{e}_x$ to the wall, $a > 0$.

   a) (**5P**) Under which (non-trivial) conditions on the system parameters does the system not radiate along the x-axis? These conditions are *not* imposed in the following.

   b) (**2P**) Determine the polarisation of the emitted radiation. In particular, what is the polarisation along the x-axis?

   c) (**5P**) We now turn to the case $\omega a \ll c$. How much energy has to be fed to the electron per cycle to maintain constant radiation strength? What is the leading multipolarity of the radiation: electric or magnetic; which "angular momentum" l? *Carefully state the reasons for your conclusion.*

3. Pressure of Solar Radiation (**6P**): The total power radiated by the sun is 3.8×10^{26} W. A portion of that hits Earth, at a distance of $r = 1.5 \times 10^{11}$ m. Thus, Earth experiences a pressure. Assume that all radiation is absorbed. (radius of the Earth $R = 6000$ km, average density $\rho = 5.5$ g cm^{-3})

   a) (**1P**) Show that the planar electro-magnetic wave (α an arbitrary, global phase)

   $$\vec{E} = A\,\vec{e}_x\;\cos[kz - \omega t - \alpha]\;\;,\;\;\vec{B} = A\,\vec{e}_y\;\cos[kz - \omega t - \alpha]\;\;,$$

   is a solution of Maxwell's equations in vacuum if $\omega = kc$.

   b) (**1P**) Confirm for this wave explicitly the relation between Poynting vector and energy-density, $\dot{\mathcal{H}} + \nabla \cdot \vec{S} = 0$. Determine the time-average of the energy density.

   c) (**2P**) Show that the time-averaged magnitude of the magnetic field of solar radiation close to Earth is 0.024 Gauß.

   d) (**2P**) Determine the time-averaged force on Earth. Set it in relation to the gravitational force on Earth, exerted by the sun.

Light travels faster than sound.
That is why people seem bright until you hear what they say.
– *Click& Clack*