Problem Sheet 1

Special due date: 1 February 2017 16:00

You can do problems 1, 2, 4 and 6 without listening to the first lecture!

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at http://home.gwu.edu/~hgrie/lectures/edyn17/edyn17.html.

1. Guesstimates (**6P**): In a "guesstimate", one attempts an *order-of-magnitude* **estimate** of a quantity or effect by educated **guesswork** and as few math as possible. The trick is to see how far one can get with what is in one's head and with the information given, without resorting to books or even calculators. Guesstimates are usually made without sribbling or on a very small piece of paper, like the proverbial "back-of-the-envelope". If you come up with numbers consisting of more than 1 significant digit, you have not understood the concept. For the following guesstimates, you may find the information on essential numbers in the syllabus useful. They also also to familiarise you with the beauty of using "natural units" for nucleon masses etc.

   I have a "rechargable Li-I battery" with "AA, 2500 mAh, 1.2 V" printed on it.

   a) (**3P**) Which fraction of the electrons inside the battery needs to be dissociated from atoms/ions to comply with the specifications?

   b) (**3P**) To get a feeling how "powerful" the battery is: How much matter could you create by converting the energy stored in a charged battery (as opposed to an uncharged one), if that were possible? A gramme? A grain of dust? An amoeba's worth? A gross of atoms? Fractions of an electron?

2. A Two-dimensional Green's Function (**5P per correct, independent way**) Show that the Green's function to the *two*-dimensional Poisson equation "without boundaries at infinity" is

$$G(\vec{r},\vec{r}') = \alpha \ln \frac{|\vec{r}-\vec{r}'|}{C} \quad ,$$

   where C is an arbitrary constant. Determine the constant α. There are several ways to do this problem, given the toolchest developed in the last semester. **Each correct way gives 5 points.**

   You might want to recall also some Green's functions of one- and three-dimensional "empty" space.

3. Magnetic Monopoles (**4P**): Let's pretend magnetic monopoles, i.e. point-like objects with a *magnetic charge* g_M, do exist.

   a) (**2P**) What would Maxwell's equations look like? Start from the modification necessary to accommodate a source for the magnetic field, and assume that magnetic charges are conserved to find modification(s) for the other equation(s).

   b) (**2P**) Calculate the magnetic field of a magnetic point-charge of strength g_M.

Please turn over.

4. Simultaneïty and Locality (**4P**): In an inertial system **I**, two events occur at different spacetime coordinates $(ct_1, x_1, 0, 0)$ and $(ct_2, 0, 0, 0)$, where $x_1 \neq 0,\ 0 \neq t_1 \neq t_2 \neq 0$.

   a) (**2P**) Under which conditions on the invariant distance between the events can we find an inertial system **II** in which the events occur *simultaneously*, or (ii) *at the same place* in space, respectively?

   b) (**2P**) Determine for both cases the relative velocity between inertial systems **I** and **II**.

5. Relativistic Photography (**7P**, plus **up to 10P extra credit**): In a NASA attempt to document the Lorentz-contraction effect, an astronaut shoots a picture of a cuboid passing the astronaut at high, constant velocity. The photograph is taken at the moment of smallest distance between astronaut and cuboid. In the rest system of the cuboid, at least one of its edges is parallel to its motion. During the following meeting of the Senate Sub-Sub-Committee of Sincere Science, a senator smells a scam: "The astronaut just took a photograph of the cuboid at rest, at a slightly tilted angle." What does the photograph show? Are the two scenarios reconcilable?

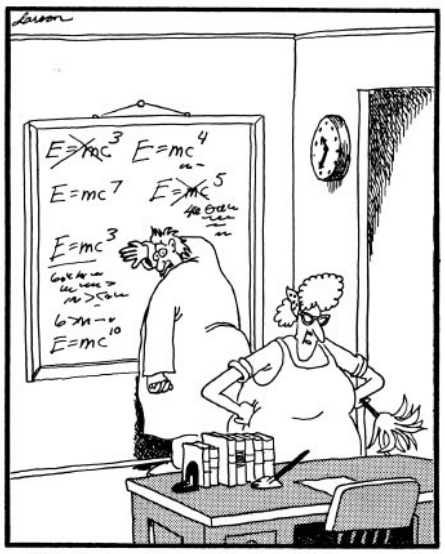
   Hint: Consider besides Lorentz-contractions also the emission times of two light-rays which enter the camera simultaneous but come from different edges.

   Hint: This is an open-ended problem. You could in addition construct the picture: when the cuboid has sides of different colour; taking into account that light is reflected not uniformly from its sides, but preferably perpendicular to the sides, etc. You could make a picture, or even a movie using Mathematica, Maple, MatLab or similar. You can get **up to 10 points of extra credit**!

6. Scalar Field and Point-Particle (**4P**): Consider the following action coupling a point-particle to a (space-time dependent) Lorentz-scalar field $\varphi(x)$ by strength g:

$$S = -mc \int \mathrm{d}s \; - g \int \mathrm{d}s \; \varphi\left(x(s)\right)$$

   Determine the equation of motion for the particle. Show that it does not resemble that of a charged particle subject to the Lorentz-force. The simplest coupling of a scalar field to a particle can therefore not reproduce Electrodynamics.



"Now that desk looks better. Everything's squared away, yessir, squaaaaaared away."

From 'Valley of the Far Side' by Gary Larson
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