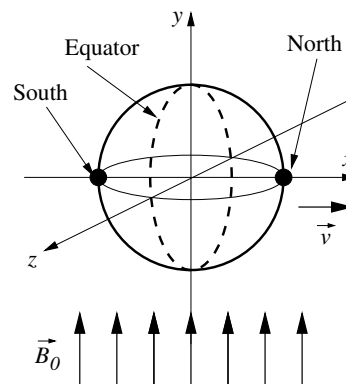


### Problems in Lecture

14 February 2017

You will develop solutions to the following problems in class. It is helpful if you have looked at them before class and have come up with a rough strategy what to do. We focus on problem 1 first, and then proceed with problem 2 or 3 if there is time.

1. RELATIVITY AND FIELDS: Take a sphere with radius  $R$  which is overall charge-neutral and perfectly conducting ( $\epsilon = \infty, \mu = 1$ ). It moves with constant, possibly relativistic velocity  $\vec{v} = v\vec{e}_x, v \ll c$ , in a uniform and constant magnetic field  $\vec{B} = B\vec{e}_y$ . The “Equator”, “North” and “South” pole of the sphere are marked as in the figure.



- a) Determine the surface charge density which an observer at rest sees induced on the “Equator”, “North” and “South” pole of the moving sphere.

**Hint:** Total scalar potential outside a grounded, perfectly conducting sphere put into a constant, external electric field  $\vec{E}_0$ :

$$\Phi(r, \theta) = -|\vec{E}_0| \cos \theta \left( r - \frac{R^3}{r^2} \right) , \text{ where } \theta \text{ is the angle between } \vec{E}_0 \text{ and } \vec{r}. \text{ Show this!}$$

- b) How big is the electric dipole moment of the moving sphere, seen by the observer at rest?
2. POINT-CHARGE INSIDE A CONDUCTING SPHERE: A point-charge  $q$  is located outside a homogeneous, grounded sphere with radius  $R$ , at a distance  $\vec{a}$  from the centre of the sphere ( $R < a$ ). It is convenient to identify the coordinate origin with the centre of the sphere.
    - a) Draw a field-line sketch of this configuration. Without doing a calculation, argue how much total surface charge is accumulated on the sphere.
    - b) Confirm that the electrostatic potential is given in terms of Legendre polynomials  $P_l$  and the angle  $\theta = \angle(\vec{r}; \vec{a})$  as

$$\Phi(\vec{r}) = \begin{cases} \frac{q}{|\vec{r} - \vec{a}|} - q R \sum_{l=0}^{\infty} \frac{(ar)^l}{(R^2)^{l+1}} P_l(\cos \theta) & \text{for } r < R \\ 0 & \text{for } r > R \end{cases} .$$

- c) Determine the induced surface charge density on the surface of the sphere.
- d) Determine the change in energy when the sphere is moved from infinity to this position, i.e. put over the charge. Your result should be analytical and not contain a series. Do you gain or have to invest energy in this process?

