Electrodynamics: Conventions

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I define and normalise special functions (like Legendre Polynomials $P_l(z)$, spherical harmonics $Y_{lm}(\vartheta, \varphi)$ etc.) as in [Brau] = [Jack] = [M] = Gradshteyn/Ryzhik: *Table of Integrals, Series and Products.*

Einstein's summation convention: sum over all indices which appear exactly once as subscript and exactly once as superscript, except if stated otherwise.

"east-coast" metric	$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$	contra-variant 4-vectors	$V^\mu = (V^0, \vec{V})$
space-time indices	$\mu, \nu, \rho, \dots = 0, \dots, 3$	spatial indices	$i, j, k, \dots = 1, 2, 3$
function of 3-dim. vector	$f(\vec{r})$	function of 4-dim. vector	$f(x^\mu) \equiv f(x)$
3-dim. scalar product	$a_i b^i \equiv a^i b_i \equiv \vec{a} \circ \vec{b} \equiv \vec{a}^T \vec{b}$	4-dim. scalar product	$a \cdot b \equiv a_\mu b^\mu \equiv a^\mu b_\mu$
Green's function of linear, d-dimensional differential operator D		$D\, G(\vec{r}, \vec{r}\,') = \delta^{(d)}(\vec{r} - \vec{r}\,')$	

Systems of Electromagnetic Units are not a glorious chapter of metrology: There exist five "major" systems, and several variants which are used next to each other or even simultaneously within the same branch of Physics. I use the **Gaussian (cgs) System**, which knows only three fundamental units (centimetres, grams and seconds) and shortens formulae (no ϵ_0 or μ_0, hardly any 4π etc.). The standard system of everyday life is SI, with the familiar basic units m, kg, s, A, K, rad and cd. While the Physics contents of an equation is of course independent of the system used, one is in applications obviously interested in questions like "at which current is this resistor blasted". [Lan2/8, Low, Schw] and [Jack, Chap. 1-9] use cgs; [Brau, Grif, Schwa] and [Jack, Chap. 10-16] use SI.

You can choose whichever system you want, but I expect your solutions to be consistent!

Fortunately, the system used is uniquely determined by any two of the fundamental equations which contain $\vec{E}$ and a combination of $\vec{E}$ and $\vec{B}$. You find below a table with two equations and conversion factors from Gaussian to SI units. More on systems, units and dimensions e.g. in the appendices of [Jack] and [Brau].

Unit	**Gaussian cgs System**	**Syst. Internat. d'Unitès SI**
el. field strength of point charge Q	$\frac{Q}{r^2}$	$\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
Lorentz force $\vec{F}$	$Q\,(\vec{E} + \frac{1}{c}\, \vec{v} \times \vec{B})$	$Q\,(\vec{E} + \vec{v} \times \vec{B})$
Gauss' law (Maxwell I)	$\vec{\nabla} \cdot \vec{E} = 4\pi\, \rho$	$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0}\, \rho$
Faraday's law of induction (Maxwell II)	$\vec{\nabla} \times \vec{E} + \frac{1}{c}\, \dot{\vec{B}} = 0$	$\vec{\nabla} \times \vec{E} + \dot{\vec{B}} = 0$
elementary charge q	$1.6022 \times 10^{-20}\, \tilde{c}\, \underbrace{\mathrm{g}^{\frac{1}{2}}\, \mathrm{cm}^{\frac{3}{2}}\, \mathrm{s}^{-1}}_{\text{electrostatic units (esu)}}$	1.6022×10^{-19} Coulomb (C)
dielectric constant of the vacuum ϵ_0	1 (no dimensions)	$\frac{10^7}{4\pi c^2}\, \frac{\mathrm{A}^2\, \mathrm{s}^2}{\mathrm{kg\ m}} = \frac{1}{\mu_0 c^2}$
permeability of the vacuum μ_0	1 (no dimensions)	$4\pi \times 10^{-7}\, \frac{\mathrm{kg\ m}}{\mathrm{A}^2\, \mathrm{s}^2}$
unit potential $(\Phi, \vec{A})$	$\frac{10^8}{\tilde{c}}\, \underbrace{\mathrm{g}^{\frac{1}{2}}\, \mathrm{cm}^{\frac{1}{2}}\, \mathrm{s}^{-1}}_{\text{statvolt=erg esu}}$	$= 1$ Volt (V)
unit el. field strength $\vec{E}$	$\frac{10^6}{\tilde{c}}\, \underbrace{\mathrm{g}^{\frac{1}{2}}\, \mathrm{cm}^{-\frac{1}{2}}\, \mathrm{s}^{-1}}_{\text{statvolt cm}^{-1}\text{=dyn esu}}$	$= 1\, \frac{\mathrm{V}}{\mathrm{m}}$
unit mag. induction $\vec{B}$	$10^4\, \underbrace{\mathrm{g}^{\frac{1}{2}}\, \mathrm{cm}^{-\frac{1}{2}}\, \mathrm{s}^{-1}}_{\text{Gauß (G)}}$	$= 1$ Tesla (T)

$c := 2.997\,924\,58 \times 10^8\ \mathrm{m\ s}^{-1}$: velocity of light in vacuum;
$\tilde{c} = 2.997\,924\,58 \times 10^{10}$: num. conversion factor: velocity of light in vacuum in Gaussian units [$\mathrm{cm\ s}^{-1}$].