GWU Department of Mathematics

Topics in Logic: Axiomatic Set Theory

(CRN 66818 Math 6720: Topics in Logic)

Fall 2015

MW 3:45–5:00pm

1957 E Street, Room 313

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Description

Naive set theory was founded by Georg Cantor who defined a set as a "collection into a whole of definite, distinct objects of our intuition or our thought." However, this definition allows the existence of some unusual sets that lead to paradoxes. The paradoxes leave set theorists with the task of determining which properties do define sets. Unfortunately, Kurt Gödel's results indicate that a complete answer to this question is not even possible. Therefore, axiomatic set theory attempts a less lofty goal. It formulates some of the relatively simple properties of sets, used by mathematicians, as axioms. Within this axiomatic system, practically all notions of contemporary mathematics can be defined and their properties can be derived. In this sense the axiomatic set theory serves as a foundation of mathematics.

Cantor proved that there are infinitely many infinities. He showed that, while there are as many rational numbers as natural numbers, there are more real numbers than natural numbers. Is there an intermediate infinity? A negative answer to this question is known as the *continuum hypothesis*. In 1963, Paul Cohen obtained a surprising result, which was "rather unsatisfactory to an average mathematician," by establishing that the continuum hypothesis is *independent*, that is, neither provable nor refutable from the usual set-theoretic axioms. Another mathematical principal which the usual set-theoretic axioms fail to settle is the *axiom of choice*. The independence results use the *forcing technique* for which Cohen won the Fields Medal.

Required background

Mathematical maturity and familiarity with the mathematical proof. Math 6720 can be taken for credit repeatedly. Advanced undergraduate students may also take this course for credit.

Textbook

Set Theory, K. Kunen, Elsevier.

Other reading material will be provided in class.

Learning goals

As a result of completing this course students should be able to:

1. Establish foundations of mathematics based on axiomatic set theory.

2. Apply axioms of Zermelo-Frankel set theory to develop familiar mathematical concepts and results;

3. Design and analyze various set-theoretic models satisfying certain axioms or their negations;

4. Discuss and establish the independence results.

Grading

Based on class participation and in-class presentation (20%), take-home assignments (60%), and the final project (20%).