Examples on Computing Present Value and Yield to Maturity

(Econ 121: Mishkin Chapter 4 Materials)

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A Useful Formula:

\[ a + a^2 + a^3 + \ldots + a^n = \frac{a - a^{n+1}}{1 - a}. \]  \hfill (1)

Special Case: When \(0 < a < 1\), and \(n \to \infty\),

\[ a + a^2 + a^3 + \ldots + a^\infty = \frac{a}{1 - a}. \]  \hfill (2)

**Example 1** Calculate the present value for the following payments:

1. $500 two years from now when the interest rate is 5%:
   \[
   PV = \frac{500}{(1 + 0.05)^2}.
   \]

2. $100 every three years for 12 years when the interest rate is 10%:
   \[
   PV = 100 \left(1 + \frac{1}{1.1}\right) + 100 \left(1 + \frac{1}{1.16}\right) + 100 \left(1 + \frac{1}{1.19}\right) + 100 \left(1 + \frac{1}{1.112}\right)
   = 100 \left[\frac{1}{1.1^3} + \frac{1}{1.1^6} + \ldots + \frac{1}{1.1^{12}}\right].
   \]

We can apply the formula in equation (1) by recognizing that \(a = \frac{1}{1.1}\) and \(n = 4\) in this case. Applying the formula, we have

\[
PV = 100 \times \frac{\frac{1}{1.1^3} - \left[\frac{1}{1.1^3}\right]^{4+1}}{1 - \frac{1}{1.1^3}} = 205.85.
\]

3. $100 every three years for 12 years when the interest rate is 10%, plus $50 bonus at the end of 12 years.
   \[
   PV = \frac{100}{(1 + 0.1)^3} + \frac{100}{(1 + 0.1)^6} + \frac{100}{(1 + 0.1)^9} + \frac{100}{(1 + 0.1)^{12}} + \frac{50}{(1 + 0.1)^{12}}
   = 221.78
   \]

**Example 2** Suppose you buy a $1000 face-value coupon bond with a coupon rate of 10%, a maturity of 4 years,
1. Suppose you purchase the bond at a price of $1000, what is the yield to maturity?

First write down the formula for yield to maturity:

\[ 1000 = \frac{1000 \times 10\%}{1 + i} + \frac{1000 \times 10\%}{(1 + i)^2} + \frac{1000 \times 10\%}{(1 + i)^3} + \frac{1000 \times 10\%}{(1 + i)^4} + \frac{1000}{(1 + i)^4} \]

\[ \Rightarrow i = 10\% \]

2. Suppose the purchase price is $800, what is the yield to maturity?

\[ 800 = \frac{1000 \times 10\%}{1 + i} + \frac{1000 \times 10\%}{(1 + i)^2} + \frac{1000 \times 10\%}{(1 + i)^3} + \frac{1000 \times 10\%}{(1 + i)^4} + \frac{1000}{(1 + i)^4} \]

\[ \Rightarrow i = 17.34\% \]

3. Suppose the purchase price is $1200, what is the yield to maturity?

\[ 1200 = \frac{1000 \times 10\%}{1 + i} + \frac{1000 \times 10\%}{(1 + i)^2} + \frac{1000 \times 10\%}{(1 + i)^3} + \frac{1000 \times 10\%}{(1 + i)^4} + \frac{1000}{(1 + i)^4} \]

\[ \Rightarrow i = 4.43\% \]